Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input:

STEP 1

Solve the linear equation $\frac{dy(x)}{dx} - y(x) = e^{2x}$:

STEP 2

Let $\mu(x) = e^{\int -1\,dx} = e^{-x}$.

Multiply both sides by $\mu(x)$:

$$e^{-x}\frac{dy(x)}{dx} - e^{-x}y(x) = e^{x}$$

STEP 3

Substitute $-e^{-x} = \frac{d}{dx}\left(e^{-x}\right)$:

$$e^{-x}\frac{dy(x)}{dx} + \frac{d}{dx}\left(e^{-x}\right)y(x) = e^{x}$$

STEP 4

Apply the reverse product rule $f\frac{dg}{dx} + g\frac{df}{dx} = \frac{d}{dx}(f\,g)$ to the left-hand side:

$$\frac{d}{dx}\left(e^{-x}y(x)\right) = e^{x}$$

STEP 5

Integrate both sides with respect to x:

$$\int \frac{d}{dx}\left(e^{-x}y(x)\right)dx = \int e^{x}\,dx$$

STEP 6

Evaluate the integrals:

$e^{-x}y(x) = e^{x} + c_1$, where c_1 is an arbitrary constant.

STEP 7

Divide both sides by $\mu(x) = e^{-x}$:

Answer:

$$y(x) = e^{x} \left(e^{x} + c_{1}\right)$$