Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input:

STEP 1

Solve $\frac{d^2 y(x)}{dx^2} - 2 \frac{d y(x)}{dx} + y(x) = e^{2x} (5x + 3)$:

STEP 2

The general solution will be the sum of the complementary solution and particular solution.

Find the complementary solution by solving $\frac{d^2 y(x)}{dx^2} - 2 \frac{d y(x)}{dx} + y(x) = 0$:

STEP 3

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) - 2 \frac{d}{dx}\left(e^{\lambda x}\right) + e^{\lambda x} = 0$$

STEP 4

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$ and $\frac{d}{dx}\left(e^{\lambda x}\right) = \lambda e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} - 2 \lambda e^{\lambda x} + e^{\lambda x} = 0$$

STEP 5

Factor out $e^{\lambda x}$:

$$\left(\lambda^2 - 2 \lambda + 1\right) e^{\lambda x} = 0$$

STEP 6

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$$\lambda^2 - 2 \lambda + 1 = 0$$

STEP 7

Factor:

$$(\lambda - 1)^2 = 0$$

STEP 8

Solve for λ:

$\lambda = 1$ or $\lambda = 1$

STEP 9

The multiplicity of the root $\lambda = 1$ is 2 which gives $y_1(x) = c_1 e^x$, $y_2(x) = c_2 e^x x$ as solutions, where c_1 and c_2 are arbitrary constants.

The general solution is the sum of the above solutions:

$y(x) = y_1(x) + y_2(x) = c_1 e^x + c_2 e^x x$

STEP 10

Determine the particular solution to $\frac{d^2 y(x)}{dx^2} - 2 \frac{dy(x)}{dx} + y(x) = e^{2x} (5x + 3)$ by the method of undetermined coefficients:

The particular solution to $\frac{d^2 y(x)}{dx^2} - 2 \frac{dy(x)}{dx} + y(x) = e^{2x} (5x + 3)$ is of the form:

$y_p(x) = a_1 e^{2x} + a_2 e^{2x} x$

STEP 11

Solve for the unknown constants a_1 and a_2:

Compute $\frac{dy_p(x)}{dx}$:

$$\frac{dy_p(x)}{dx} = \frac{d}{dx}(a_1 e^{2x} + a_2 e^{2x} x)$$
$$= 2 e^{2x} a_1 + e^{2x} a_2 + 2 e^{2x} x a_2$$

STEP 12

Compute $\frac{d^2 y_p(x)}{dx^2}$:

$$\frac{d^2 y_p(x)}{dx^2} = \frac{d^2}{dx^2}(a_1 e^{2x} + a_2 e^{2x} x)$$
$$= 4 e^{2x} a_1 + (4 e^{2x} + 4 e^{2x} x) a_2$$

STEP 13

Substitute the particular solution $y_p(x)$ into the differential equation:

$$\frac{d^2 y_p(x)}{dx^2} - 2 \frac{dy_p(x)}{dx} + y_p(x) = 3 e^{2x} + 5 e^{2x} x$$

$$4 a_1 e^{2x} + a_2 (4 e^{2x} + 4 e^{2x} x) - 2 (2 a_1 e^{2x} + a_2 e^{2x} + 2 a_2 e^{2x} x) + a_1 e^{2x} + a_2 e^{2x} x = 3 e^{2x} + 5 e^{2x} x$$

STEP 14

Simplify:

$(a_1 + 2 a_2) e^{2x} + a_2 e^{2x} x = 3 e^{2x} + 5 e^{2x} x$

STEP 15

Equate the coefficients of e^{2x} on both sides of the equation:

$a_1 + 2 a_2 = 3$

STEP 16

Equate the coefficients of $e^{2x} x$ on both sides of the equation:

$a_2 = 5$

STEP 17

Solve the system:

$a_1 = -7$
$a_2 = 5$

STEP 18

Substitute a_1 and a_2 into $y_p(x) = e^{2x} a_1 + e^{2x} x a_2$:

$y_p(x) = -7 e^{2x} + 5 e^{2x} x$

STEP 19

The general solution is:

Answer:

$$y(x) = y_c(x) + y_p(x) = -7 e^{2x} + 5 e^{2x} x + c_1 e^x + c_2 e^x x$$