Restricciones de ≥

$$\textbf{MAX:} \quad Z = 4x_1 + 3x_2$$

$$\begin{cases} 6x_1 + 16x_2 \text{ £ } 48.000 \\ 12x_1 + 6x_2 \text{ £ } 42.000 \\ x_2 \text{ ³ } 1.500 \end{cases}$$

$$x_1, x_2 \text{ ³ } 0$$

Forma estándar

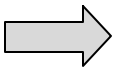
$$\textbf{MAX:} \quad Z = 4x_1 + 3x_2$$

$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48.000 \\ 12x_1 + 6x_2 + x_4 = 42.000 \\ x_2 - x_5 = 1.500 \end{cases}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

PRIMERA BASE DEL “SIMPLEX”

$$\begin{cases} x_3 = 48.000 \\ x_4 = 42.000 \\ -x_5 = 1.500 \end{cases}$$



VULNERA EL PRINCIPIO DE NN

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

PRIMERA BASE DEL “SIMPLEX”

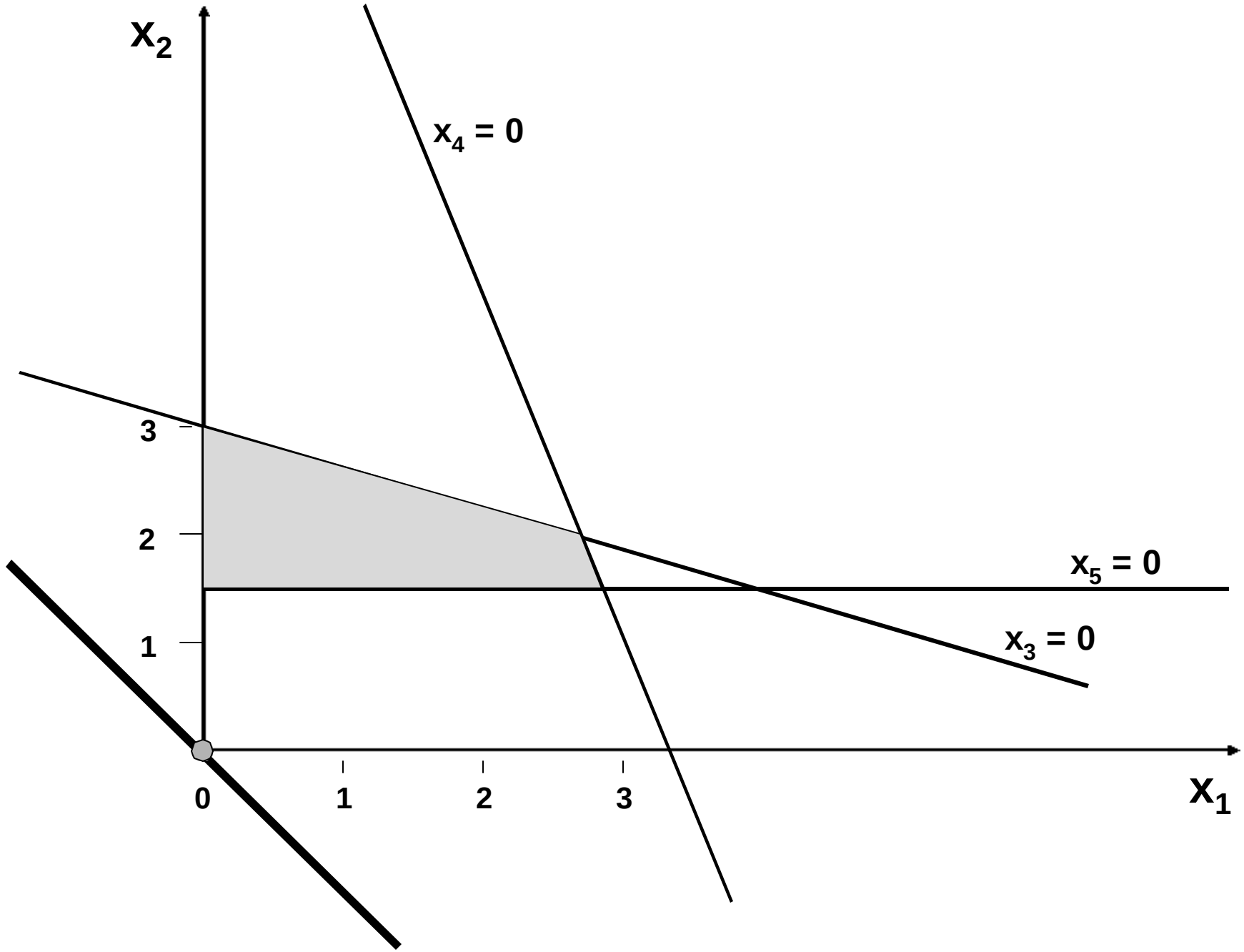
MAX: $Z = 4\,x_1 + 3\,x_2 - M \cdot \mu_1$

$$\begin{cases} x_3 = 48.000 \\ x_4 = 42.000 \\ -x_5 + \mu_1 = 1.500 \end{cases}$$

$$x_1, x_2, x_3, x_4, x_5, \mu_1 \geq 0$$

PROBLEMA DE MINIMIZACIÓN

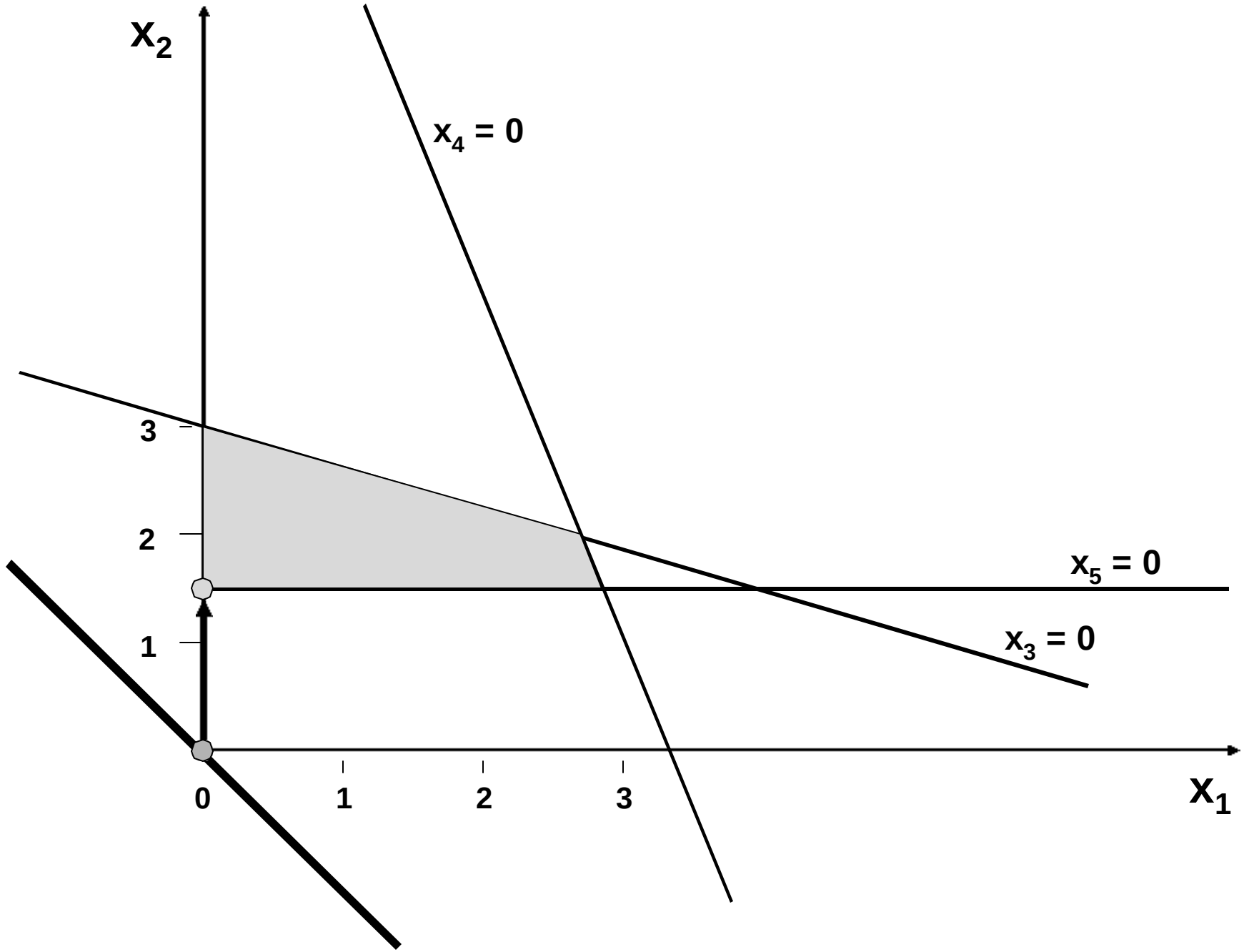
MIN: $Z = \ldots \mathbf{+ M \cdot \mu_1} \ \ldots$


x_2
$x_4 = 0$
3
2
1
$x_5 = 0$
$x_3 = 0$
0
1
2
3
x_1

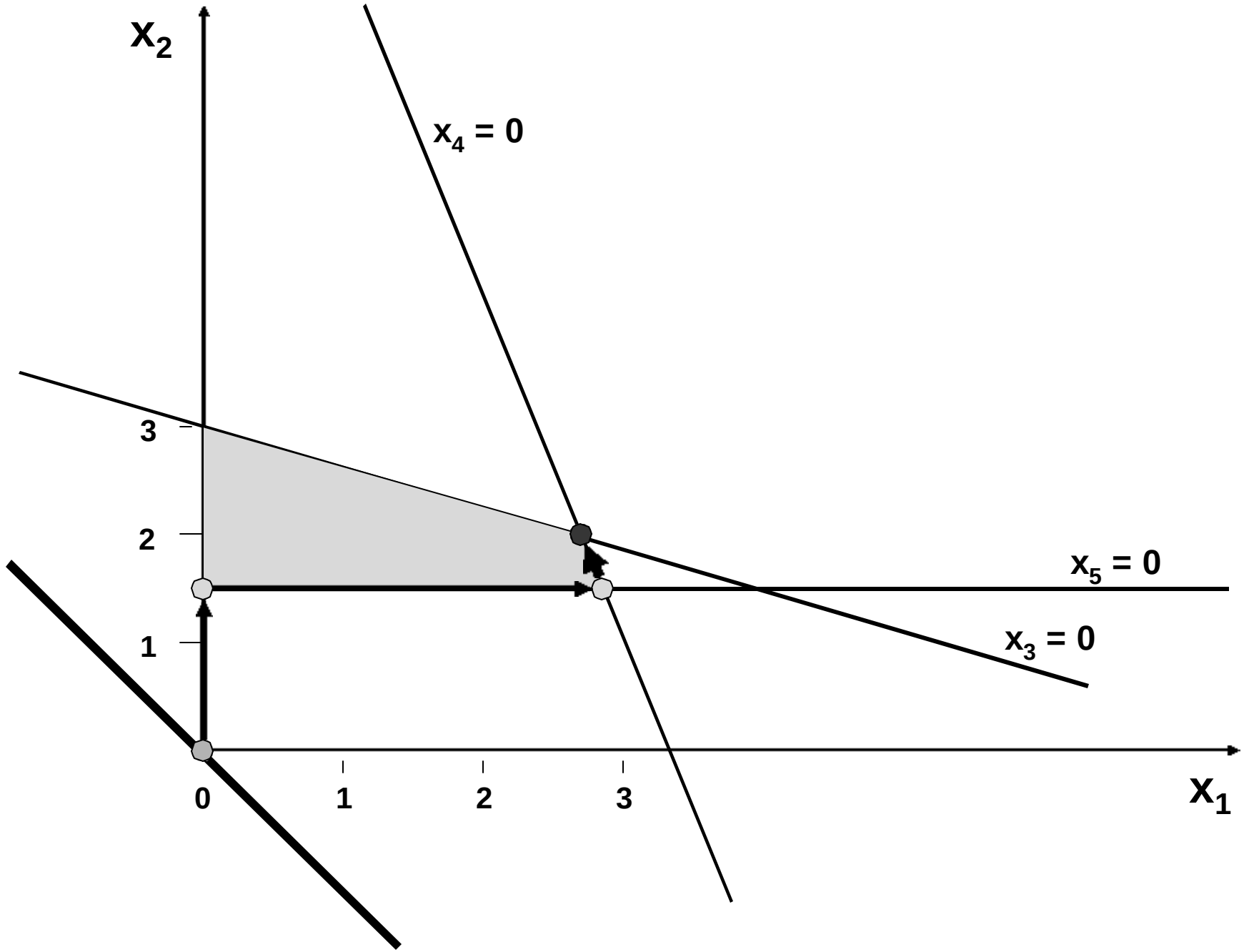
		c_j	4	3	0	0	0	-M	
$\mathbf{c_k}$	$\mathbf{x_k}$	**B**	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$	$\mathbf{A_5}$	$\mathbf{A_\mu}$	b_i/a_{ij}
0	$\mathbf{x_3}$	48.000	6	16	1				3.000
0	$\mathbf{x_4}$	42.000	12	6		1			7.000
⇦ -M	**μ**	1.500		(1)			-1	1	1.500
Z = -1.500 M			-4	-M-3	0	0	M	0	

⇧

		c_j	4	3	0	0	0	-M	
$\mathbf{c_k}$	$\mathbf{x_k}$	**B**	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$	$\mathbf{A_5}$	$\mathbf{A_\mu}$	b_i/a_{ij}
0	$\mathbf{x_3}$	24.000	6		1		16	-16	4.000
0	$\mathbf{x_4}$	33.000	12			1	6	-6	2.750
3	$\mathbf{x_2}$	1.500		1			-1	1	---
Z = 4.500			-4	0	0	0	-3	3+M	


x_2
$x_4 = 0$
3
2
1
$x_5 = 0$
$x_3 = 0$
0
1
2
3
x_1

		c_j	4	3	0	0	0	-M	
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5	A_μ	b_i/a_{ij}
0	x_3	24.000	6		1		16	-16	4.000
⇐ 0	x_4	33.000	(12)			1	6	-6	2.750
3	x_2	1.500		1			-1	1	---
Z = 4.500			-4	0	0	0	-3	3+M	
			⇑						
⇐ 0	x_3	576,92			1	-0,5	(13)	-13	576,9231
4	x_1	5.500	1			0,0833	0,5	-0,5	5.500
3	x_2	1.500		1		0	-1	1	---
Z = 15.500			0	0	0	0,3333	-1	1+M	
							⇑		
0	x_5	576,9231			0,0769	-0,0385	1	-1	
4	x_1	2.461,539	1		-0,0385	0,1026			
3	x_2	2076,923		1	0,0769	-0,0385			
Z = 16.076,92			0	0	0,0769	0,2949	0	M	


x2
x4 = 0
3
2
1
x5 = 0
x3 = 0
0
1
2
3
x1

Restricciones de =

MAX: $Z = 4x_1 + 3x_2$

$$\begin{cases} 6x_1 + 16x_2 \leq 48.000 \\ 12x_1 + 6x_2 \leq 42.000 \\ x_2 = 1.500 \end{cases}$$

$$x_1, x_2 \geq 0$$

Forma estándar

MAX: $Z = 4x_1 + 3x_2$

$$\begin{cases} 6x_1 + 16x_2 + x_3 = 48.000 \\ 12x_1 + 6x_2 + x_4 = 42.000 \\ x_2 = 1.500 \end{cases}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

PRIMERA BASE DEL “SIMPLEX”

$$\begin{cases} x_3 = 48.000 \\ x_4 = 42.000 \\ 0 = 1.500 \end{cases} \Rightarrow$$

NO ES COMPATIBLE

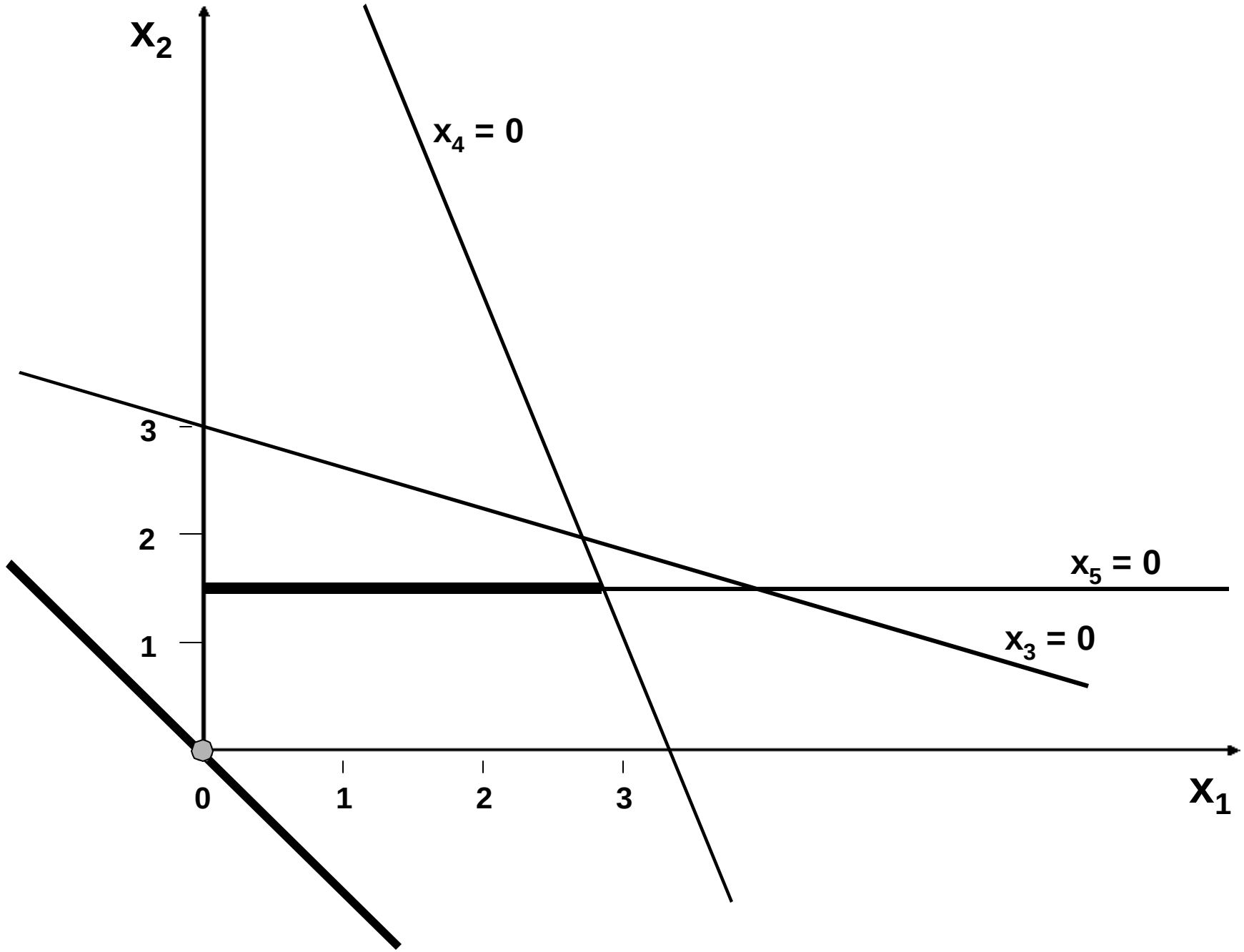
$$x_1, x_2, x_3, x_4 \geq 0$$

PRIMERA BASE DEL “SIMPLEX”

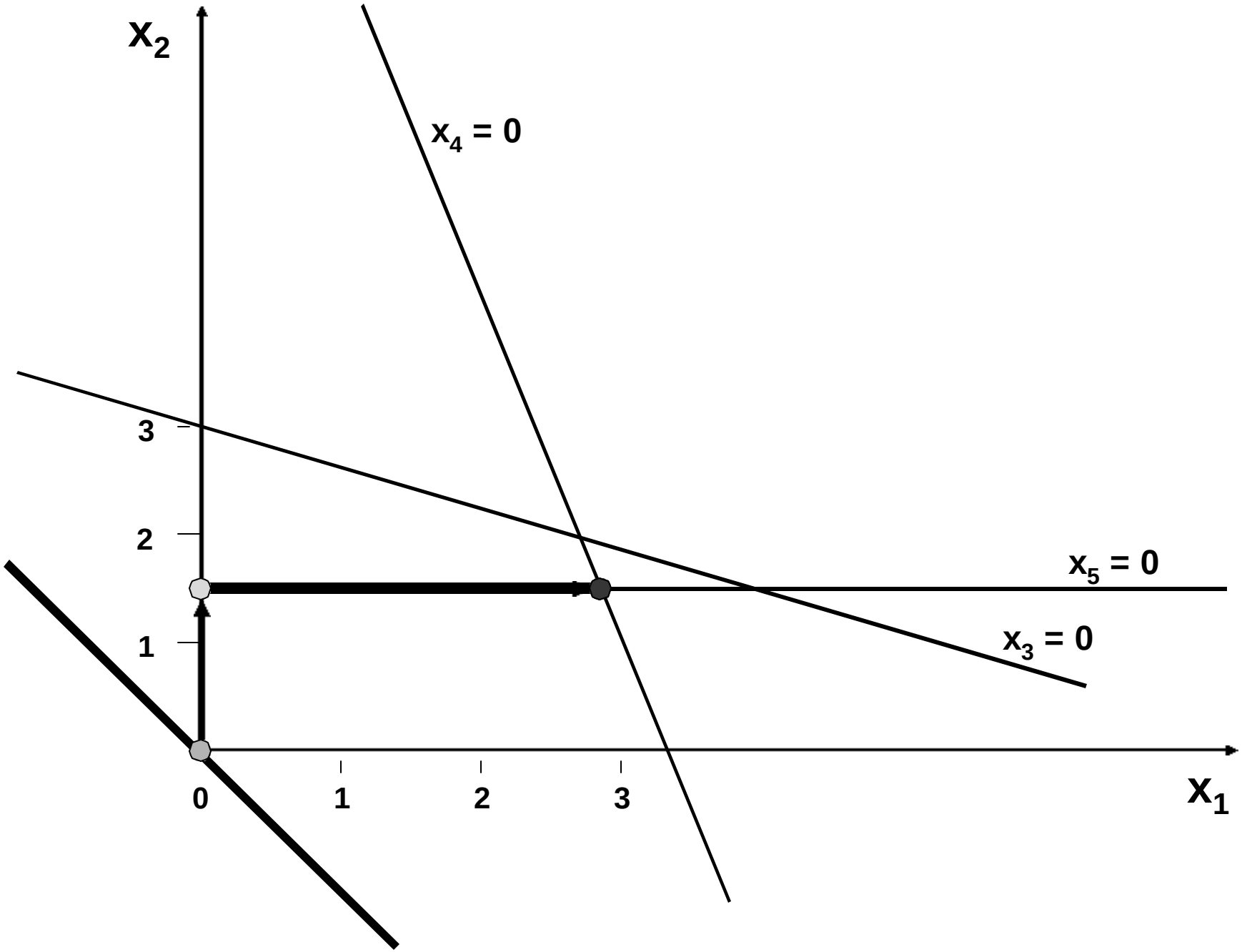
MAX: $Z = 4\,x_1 + 3\,x_2 - M\,\ddot{e}_1$

$$\begin{cases} x_3 = 48.000 \\ x_4 = 42.000 \\ +\,\ddot{e}_1 = 1.500 \end{cases}$$

$$x_1, x_2, x_3, x_4, \ddot{e}_1 \geq 0$$


x_2
$x_4 = 0$
3
2
1
$x_5 = 0$
$x_3 = 0$
0
1
2
3
x_1

		c_j	4	3	0	0	-M	
c_k	x_k	B	A_1	A_2	A_3	A_4	$A_ë$	b_i/a_{ij}
0	x_3	48.000	6	16	1			3.000
0	x_4	42.000	12	6		1		7.000
⇐ -M	ë	1.500		(1)			1	1.500
Z = - M 1.500			-4	-M-3	0	0	3+M	
0	x_3	24.000	6		1		16	4.000
⇐ 0	x_4	33.000	(12)			1	6	2.750
3	x_2	1.500		1			-1	---
Z = 4.500			-4	0	0	0	-3+M	
0	x_3	7.500			1	-0,5	-13	
4	x_1	2.750	1			0,0833	-0,5	
3	x_2	1.500		1		0	1	
Z = 15.500			0	0	0	0,3333	1+M	


x_2
$x_4 = 0$
3
2
1
$x_5 = 0$
$x_3 = 0$
0
1
2
3
x_1

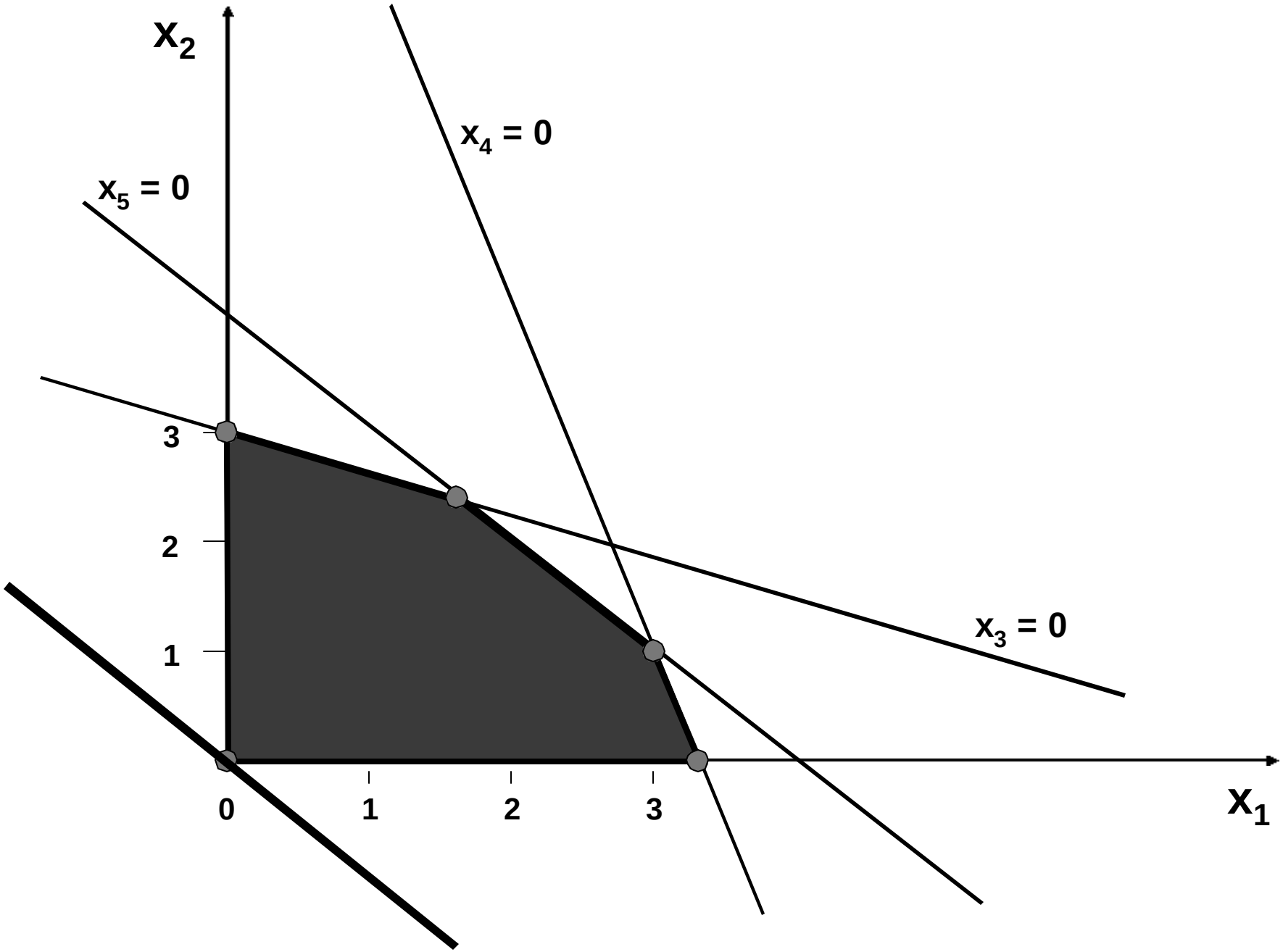
CASOS PARTICULARES

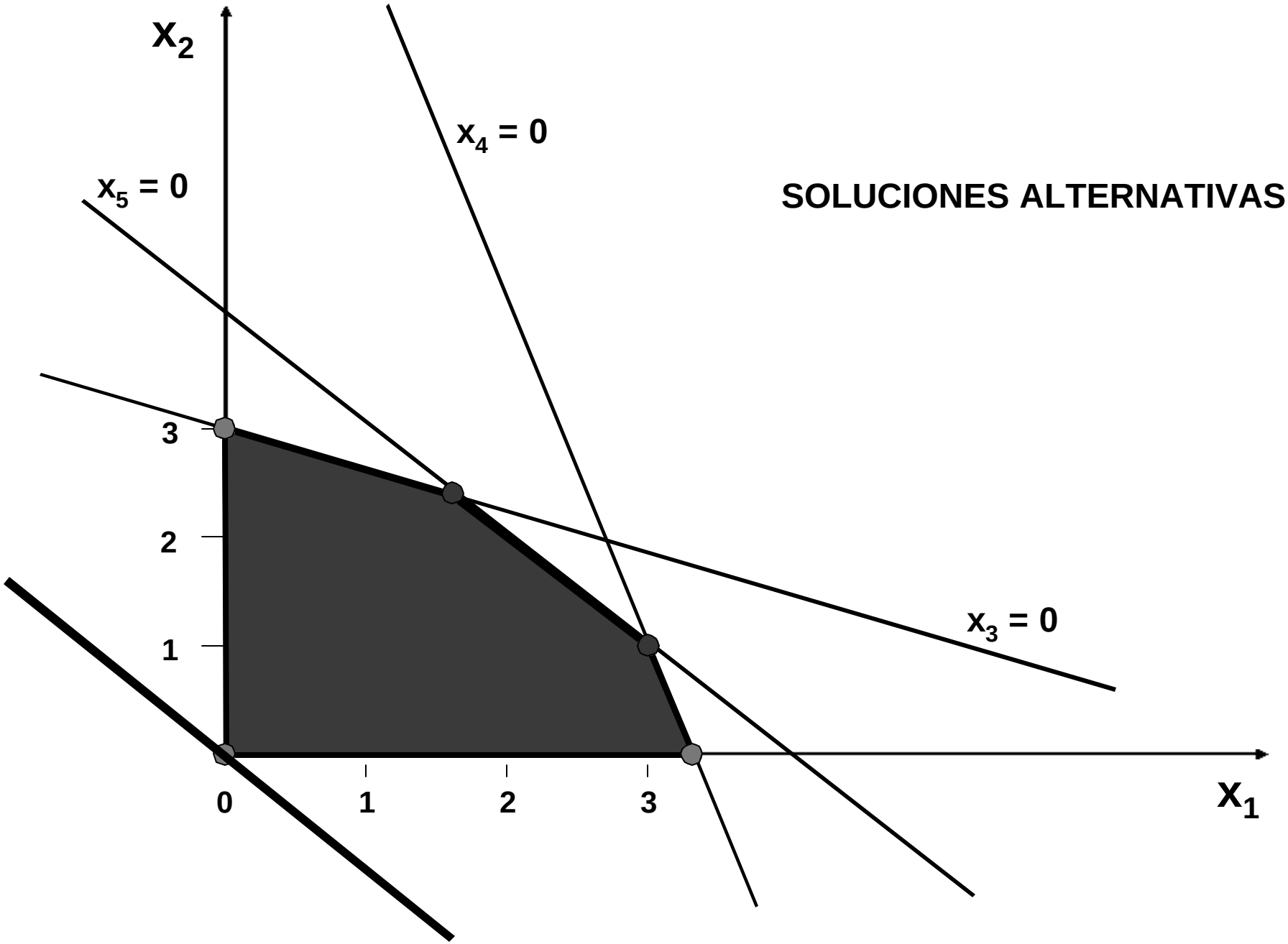
- SOLUCIÓN ALTERNATIVA
- SOLUCIÓN DEGENERADA
- POLITOPO ABIERTO
- SOLUCIÓN INCOMPATIBLE

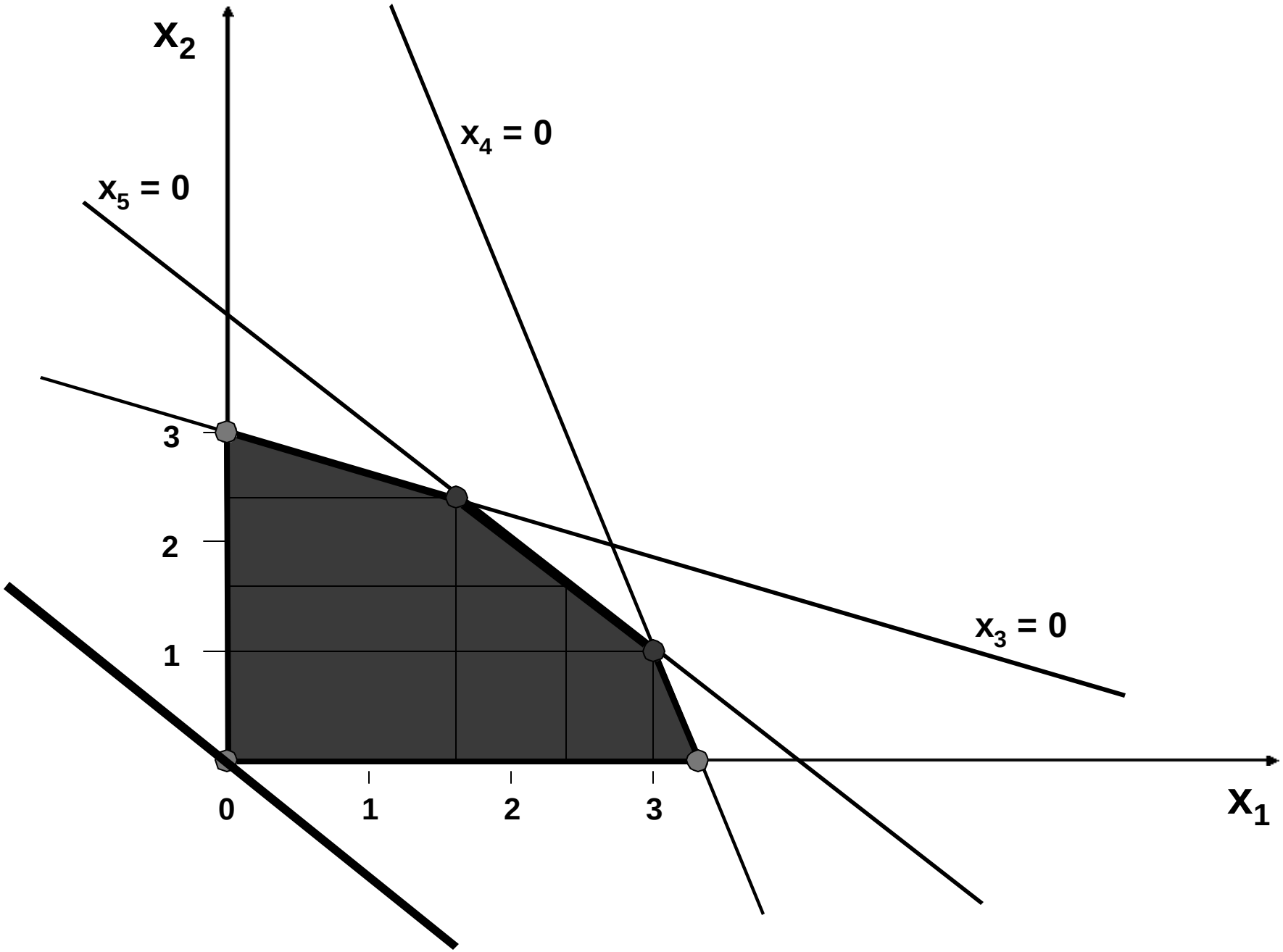
$$\text{MAX:} \quad Z = 3x_1 + 3x_2$$

$$\begin{cases} 6x_1 + 16x_2 \leq 48.000 \\ 12x_1 + 6x_2 \leq 42.000 \\ 9x_1 + 9x_2 \leq 36.000 \end{cases}$$

$$x_1, x_2 \geq 0$$


x_2
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
3
2
1
0
1
2
3
x_1


x_2
$x_4 = 0$
$x_5 = 0$
SOLUCIONES ALTERNATIVAS
3
2
1
$x_3 = 0$
0
1
2
3
x_1


x2
x4 = 0
x5 = 0
3
2
1
x3 = 0
0
1
2
3
x1

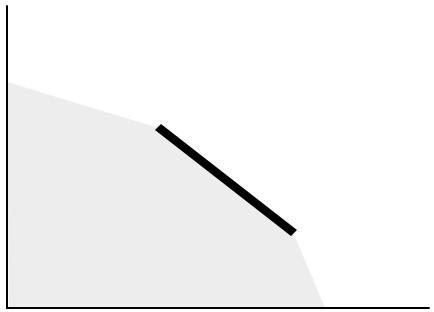
		c_j	3	3	0	0	0	
c_k	x_k	**B**	A_1	A_2	A_3	A_4	A_5	b_i/a_{ij}
0	x_3	14.000			1	5/3	-26/9	8.400
3	x_1	3.000	1			1/6	-1/9	18.000
3	x_2	1.000		1		-1/6	2/9	---
Z = 12.000			0	0	0	**0***	3/9	

		c_j	3	3	0	0	0	
$\mathbf{c_k}$	$\mathbf{x_k}$	**B**	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$	$\mathbf{A_5}$	b_i/a_{ij}
0	$\mathbf{x_3}$	14.000			1	5/3	-26/9	8.400
3	$\mathbf{x_1}$	3.000	1			1/6	-1/9	18.000
3	$\mathbf{x_2}$	1.000		1		-1/6	2/9	---
Z = 12.000			0	0	0	**0***	3/9	

0	$\mathbf{x_4}$	8.400			3/5	1	-26/15	
3	$\mathbf{x_1}$	1.600	1		-1/10		16/90	
3	$\mathbf{x_2}$	2.400		1	1/10		-1/15	
Z = 12.000			0	0	**0***	0	1/3	

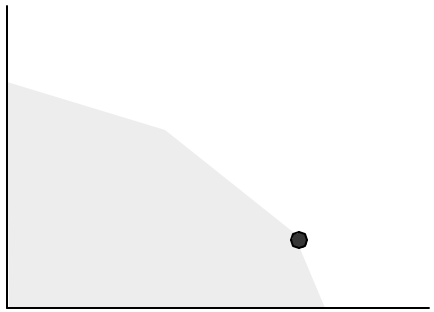
$$X = a \cdot \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix} + (1 - a) \cdot \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix}$$

$$0 \leq \alpha \leq 1$$



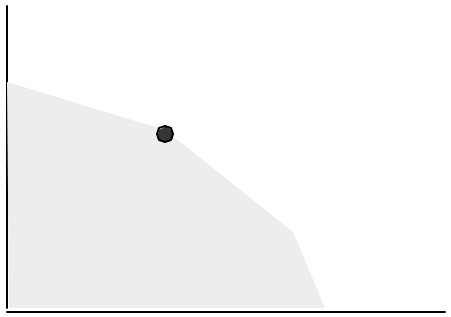
Ejemplo: $\alpha = 1$

$$X = 1 \cdot \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix}$$



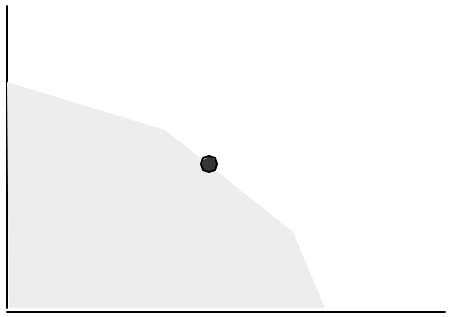
Ejemplo: $\alpha = 0$

$$X = 0 \cdot \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix}$$



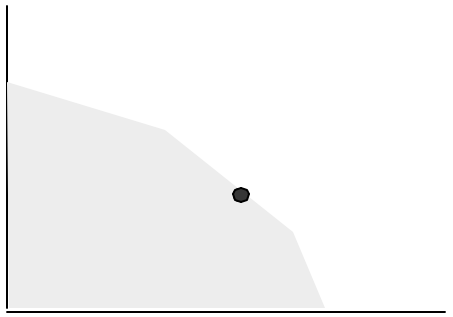
Ejemplo: $\alpha = 0{,}3$

$$X = 0{,}3 \cdot \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix} + 0{,}7 \cdot \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix} = \begin{pmatrix} 2.020 \\ 1.980 \\ 4.200 \\ 0 \\ 0 \end{pmatrix}$$



Ejemplo: $\alpha = 0{,}5$

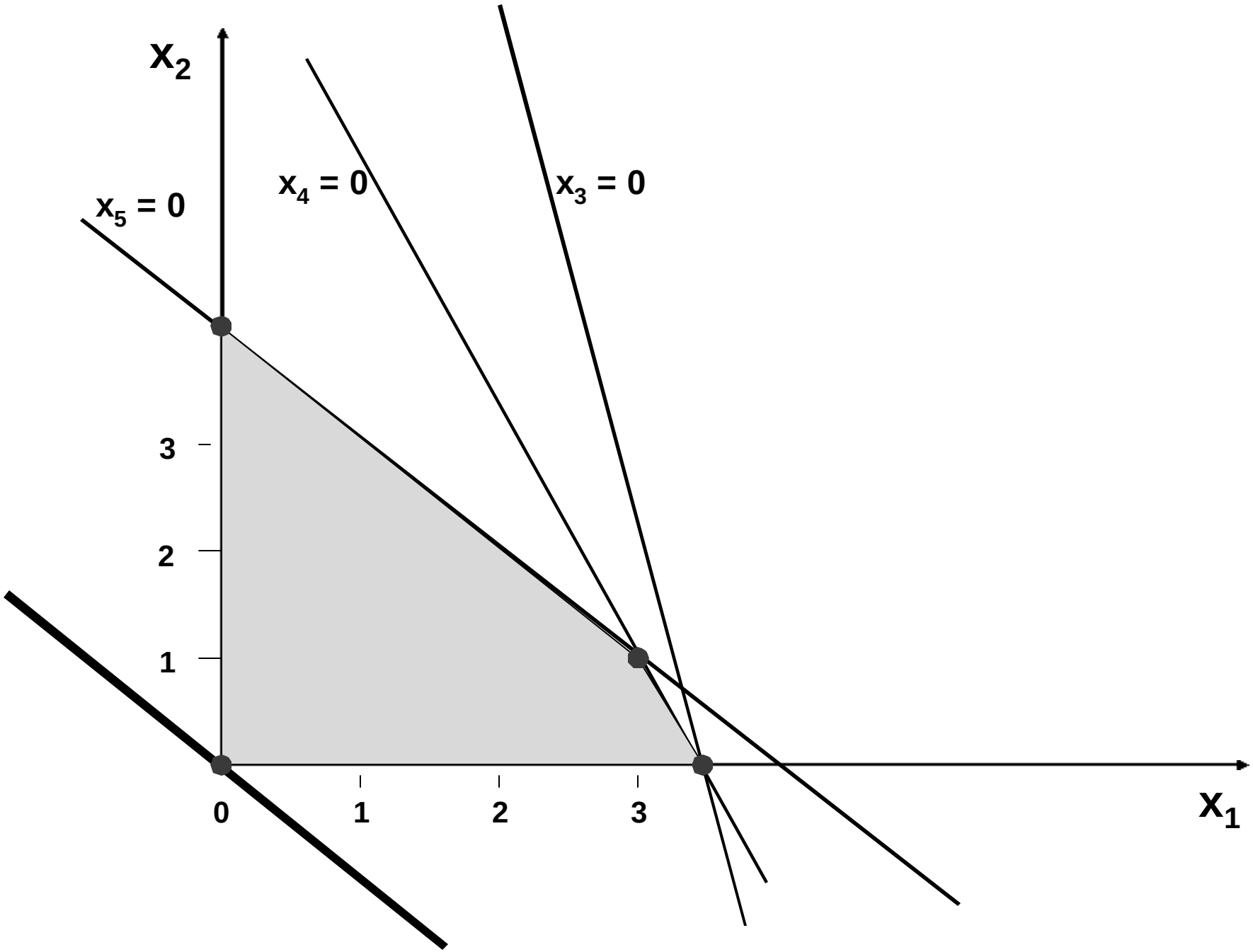
$$X = 0{,}5 \cdot \begin{pmatrix} 3.000 \\ 1.000 \\ 14.000 \\ 0 \\ 0 \end{pmatrix} + 0{,}5 \cdot \begin{pmatrix} 1.600 \\ 2.400 \\ 0 \\ 8.400 \\ 0 \end{pmatrix} = \begin{pmatrix} 2.300 \\ 1.700 \\ 7.000 \\ 0 \\ 0 \end{pmatrix}$$



$$\text{MAX:} \quad Z = 4x_1 + 3x_2$$

$$\begin{cases} 10x_1 + 4x_2 \leq 35.000 \\ 12x_1 + 6x_2 \leq 48.000 \\ 9x_1 + 9x_2 \leq 36.000 \end{cases}$$

$$x_1, x_2 \geq 0$$


x2
x4 = 0
x3 = 0
x5 = 0
3
2
1
0
1
2
3
x1

		c_j	4	3	0	0	0	
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5	b_i/a_{ij}
0	x_3	35.000	10	4	1			3.500
0	x_4	42.000	12	6		1		3.500
0	x_5	36.000	9	9			1	4.000
Z = 0			-4	-3	0	0	0	

EMPATE DE È

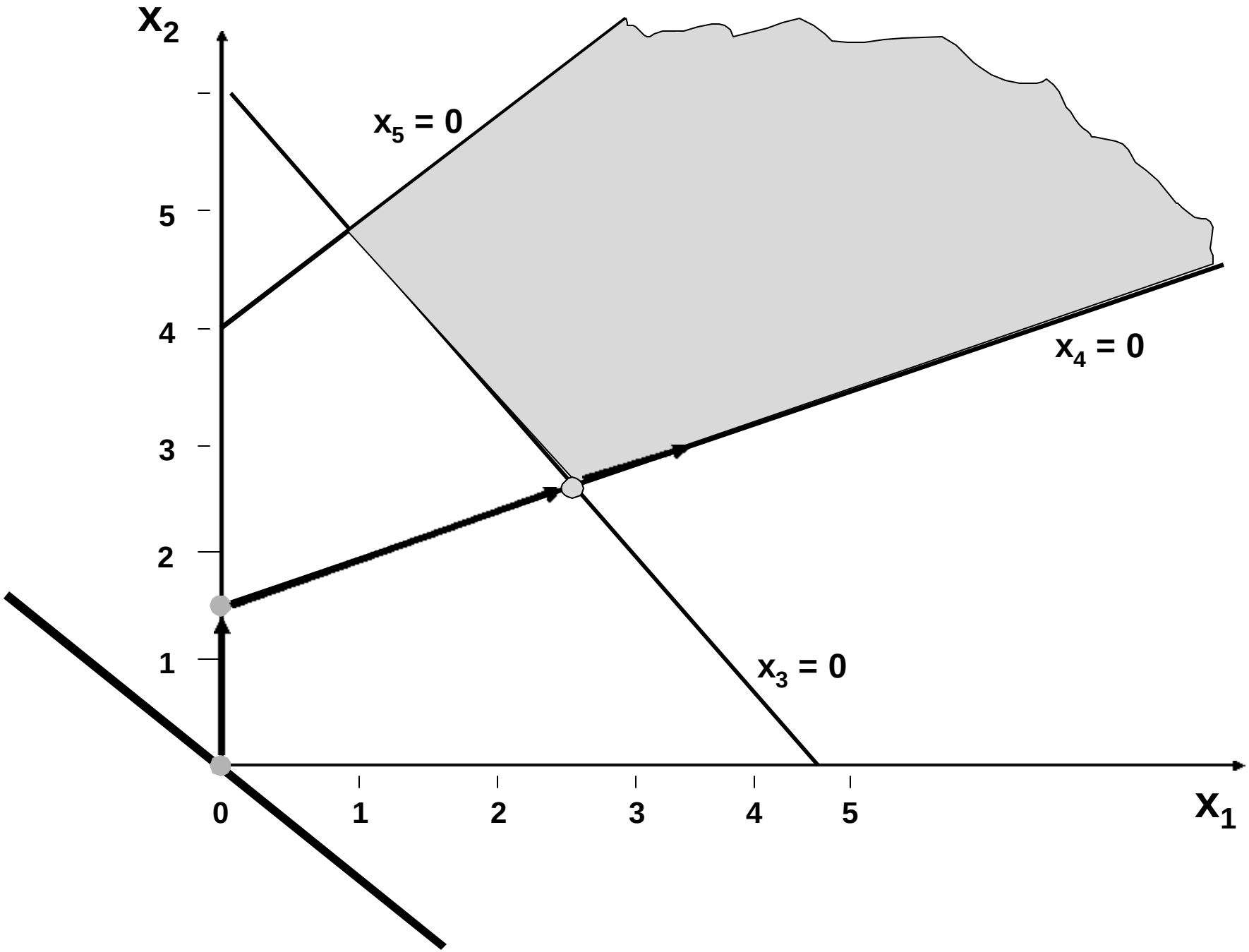
		c_j	4	3	0	0	0	
c_k	x_k	**B**	A_1	A_2	A_3	A_4	A_5	b_i/a_{ij}
4	x_1	3.500	1	0,4	0,1			8.750
0	x_4	0		1,2	-1,2	1		0
0	x_5	4.500		5,4	-0,9		1	833,33
Z = 14.000			0	-1,4	0,4	0	0	
4	x_1	3.500	1		0,5	**-0,333**		7.000
3	x_2	0		1	-1	**0,8333**		0
0	x_5	4.500			4,5	-4,5	1	1.000
Z = 14.000			0	0	-1	**1,1667**	0	
4	x_1	3000	1			**0,1667**	**-0,111**	
3	x_2	1.000		1		**-0,1667**	**0,2222**	
0	x_3	1.000			1	-1	**0,2222**	
Z = 15.000			0	0	0	**0,1667**	**0,2222**	

POLITOPO ABIERTO

$$\mathbf{MAX:} \quad \mathbf{Z = 4\,x_1 + 3\,x_2}$$

$$\begin{cases} \mathbf{10\,x_1 + 8\,x_2 \ \geq \ 48.000} \\ \mathbf{-4\,x_1 + 10\,x_2 \ \geq \ 16.000} \\ \mathbf{-9\,x_1 + 9\,x_2 \ \leq \ 36.000} \end{cases}$$

$$\mathbf{x_1, x_2 \ \geq \ 0}$$


x_2
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
5
4
3
2
1
0
1
2
3
4
5
x_1

3a. TABLA

		c_j	4	3	0	0	0	
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5	b_i/a_{ij}
4	x_1	2.666,67	1		-0,0758	0,0606		---
3	x_2	2.666,67		1	-0,0303	-0,0758		---
0	x_5	36.000			-0,4091	1,2327	1	---
Z = 18.666,67			0	0	-0,3939	0,0152	0	

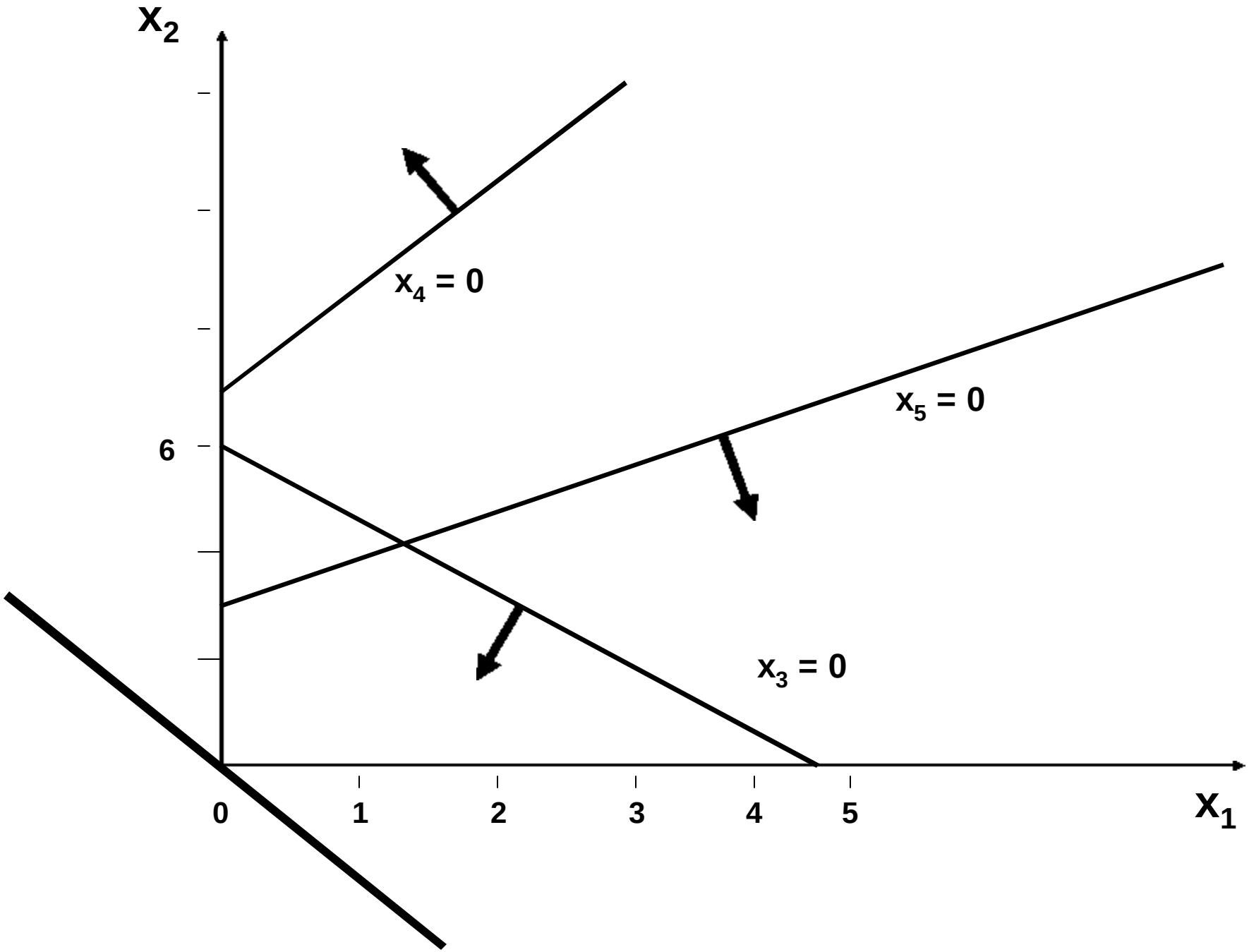
NO HAY NINGÚN È

SOLUCIÓN INCOMPATIBLE

$$\textbf{MAX:} \quad Z = 4x_1 + 3x_2$$

$$\begin{cases} 10x_1 + 8x_2 \geq 48.000 \\ -12x_1 + 6x_2 \geq 42.000 \\ -9x_1 + 9x_2 \leq 36.000 \end{cases}$$

$$x_1, x_2 \geq 0$$


x_2
6
$x_4 = 0$
$x_5 = 0$
$x_3 = 0$
0
1
2
3
4
5
x_1

3a. TABLA

		c_j	4	3				- M	- M
c_k	x_k	B	A_1	A_2	A_3	A_4	A_5	$A_{\mu 1}$	$A_{\mu 2}$
4	x_1	888,89	1		-0,0556		-0,0494	0,0556	
-M	μ_2	23.333,33			-0,3333	-1	-0,9630	0,3333	1
3	x_2	4890		1	-0,0556		0,0617	0,0556	
Z =23.333,33M + 18.222,22			0	0	-0,3889 + 0,333 M	M	-0,0123 + 0,968 M	0,3889 + 0,6667 M	0

VARIABLE ARTIFICIAL EN LA BASE

TODOS LOS z_j-c_j POSITIVOS