Mecánica de Fluidos : Guía Teórica N°1

Problema N°1: Hallar $\frac{\nu}{\nu_0} = f(P, P_0, T, T_0)$

Consideraciones previas: -) considerar la ecuación de Sutherland
-) considerar ley de gases ideales
-) Estado de referencia inicial marcado con "o": P_0, T_0, ν_0

Resolución:

Ec. Sutherland: $$\boxed{\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \cdot \left(\frac{T_0 + S}{T + S}\right)} \quad (1)$$

Ec. Gases Ideales: $$\boxed{P \cdot V = n \cdot R \cdot T} \quad (2)$$

donde: μ: viscosidad cinemática
S: constante de Sutherland
T: temperatura
n: número de moles
R: constante de los gases
P: presión
V: volumen.

Definición: $$\boxed{\nu = \frac{\mu}{\delta}} \quad (3)$$

δ: densidad
ν: viscosidad dinámica.

-) Se busca: $\nu / \nu_0 = \frac{\mu}{\delta} : \frac{\mu_0}{\delta_0} \longrightarrow \boxed{\frac{\nu}{\nu_0} = \frac{\mu \cdot \delta_0}{\mu_0 \cdot \delta}} \quad (4)$

-) La densidad δ se define:

$$\boxed{\delta = \frac{m \longrightarrow \text{masa}}{V \longrightarrow \text{volumen}}} \quad (5);$$

de (2): $P \cdot V = \frac{m}{M} \cdot R \cdot T$

$\frac{P \cdot V \cdot M}{R \cdot T} = m$; M: masa molar

queda: $\delta = \frac{m}{V} = P \cdot \frac{M}{R \cdot T}$, donde $\frac{M}{R}$ = constante

$$\boxed{\delta = P \cdot \frac{M}{R \cdot T}} \quad (6); \quad \text{para} \quad \boxed{\delta_0 = P_0 \cdot \frac{M}{R \cdot T_0}} \quad (7)$$

$$\frac{\delta_0}{\delta} = \frac{P_0 \cdot M}{R \cdot T_0} : \frac{P\,M}{R \cdot T} = \frac{P_0 \cdot \cancel{M} \cdot \cancel{R} \cdot T}{P \cdot \cancel{M} \cdot \cancel{R} \cdot T_0} ; \quad \boxed{\frac{\delta_0}{\delta} = \frac{P_0}{T_0} \cdot \frac{T}{P}} \quad (9)$$

De (8), (1) e (4):

$$\frac{\nu}{\nu_0} = \frac{\mu}{\mu_0} \cdot \frac{\delta_0}{\delta} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \cdot \left(\frac{T_0 + S}{T + S}\right) \cdot \frac{P_0}{P} \cdot \frac{T}{T_0} ;$$

$$\frac{\nu}{\nu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}+1} \cdot \left(\frac{P_0}{P}\right) \cdot \left(\frac{T_0 + S}{T + S}\right) ;$$

$$\boxed{\frac{\nu}{\nu_0} = \left(\frac{T}{T_0}\right)^{\frac{5}{2}} \cdot \left(\frac{P_0}{P}\right) \cdot \left(\frac{T_0 + S}{T + S}\right)}$$

Otra forma:

Problema 1

Hallar $\boxed{\dfrac{\nu}{\nu_0} = f(P, P_0, T, T_0)}$ (1)

Ec. Sutherland:

$$\boxed{\mu = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \cdot \left(\frac{1+K\cdot T_0}{1+KT}\right)} \quad (2)$$

Ec. Gases ideales

$$\boxed{P \cdot V = n R T} \quad (3) \longrightarrow \begin{cases} nR = \dfrac{P\cdot V}{T} \\[2ex] nR = \dfrac{P_0 \cdot V_0}{T_0} \end{cases}$$

$$\nu = \frac{\mu}{\delta} \longrightarrow \frac{\nu}{\nu_0} = \frac{\mu}{\delta} : \frac{\mu_0}{\delta_0} = \frac{\mu \cdot \delta_0}{\mu_0 \cdot \delta}$$

$$\boxed{\frac{\nu}{\nu_0} = \frac{\mu}{\mu_0} \cdot \frac{\delta_0}{\delta}} \quad (4)$$

dato; $\delta = \dfrac{m}{V}$

$\delta = \dfrac{R\cdot T}{P}$

$$\frac{\delta_0}{\delta} = \frac{R\cdot T_0}{P_0} \, \frac{P}{R\cdot T} \; , \quad \boxed{\frac{\delta_0}{\delta} = \frac{P\cdot T_0}{P_0 \cdot T}} \quad (5)$$

de (2), (5) a (4):

$$\frac{\nu}{\nu_0} = \left(\frac{T}{T_0}\right)^{3/2} \cdot \left(\frac{1+K\cdot T_0}{1+KT}\right) \cdot \frac{P}{P_0} \cdot \frac{T_0}{T}$$

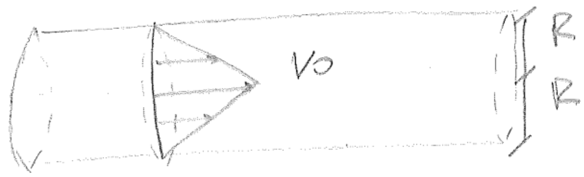
$$\boxed{\frac{\nu}{\nu_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{2}} \cdot \left(\frac{1+K\cdot T_0}{1+KT}\right) \cdot \frac{P}{P_0}}$$

Problema 2

a) Hallar Vmedia/ Vese

Datos: $\boxed{V(r) = V_0\left(1-\frac{r}{R}\right)}$ (1)

Esquema:



Consideraciones previas:) Se define Velocidad media Vm:

$$\boxed{V_m = \frac{Q}{A}} \quad (2)$$

Para hallar Q y A de (2) debemos integrar

) Tubo de sección circular.

Resolución: de (2): $\boxed{dQ = V \cdot dA}$ (3), para área circular:

$$A = \pi \cdot r^2 \longrightarrow \boxed{dA = r\, d\theta\, dr} \quad (4)$$

de (4) y (3):

$$dQ = V \cdot dA \quad : \quad \text{integro: } A \begin{cases} \theta \in [0, 2\pi] \\ r \in [0, R] \end{cases}$$

$$Q = \int_0^Q dQ = \int_0^{2\pi}\int_0^R V(r) \cdot r \cdot dr \cdot d\theta \Rightarrow$$

$$\Rightarrow Q = \int_0^{2\pi}\int_0^R V_0 \cdot \left(1 - \frac{r}{R}\right) \cdot r \cdot dr\, d\theta \Rightarrow$$

$$\Rightarrow Q = V_0\left[\int_0^{2\pi}\int_0^R \left(r - \frac{r^2}{R}\right) dr\, d\theta\right] \Rightarrow$$

$$\Rightarrow Q = V_0 \left[\int_0^{2\pi} \left(\frac{r^2}{2} \Big]_0^R - \frac{r^3}{3R} \Big]_0^R \right) d\theta \right] \Rightarrow$$

$$\Rightarrow Q = V_0 \left[\int_0^{2\pi} \left(\frac{R^2}{2} - \frac{R^3}{3R} \right) d\theta \right] \Rightarrow$$

$$\Rightarrow Q = V_0 \left[\int_0^{2\pi} \left(\frac{R^2}{2} - \frac{R^2}{3} \right) d\theta \right] \Rightarrow$$

$$\Rightarrow Q = V_0 \cdot \int_0^{2\pi} \frac{R^2}{6} \, d\theta \Rightarrow Q = V_0 \cdot \frac{2\pi}{6} \cdot R^2$$

$$\boxed{Q = V_0 \cdot R^2 \cdot \pi/3} \quad (3^*)$$

·) Calculo A. $A = \int_0^{2\pi} \int_0^R r \, dr \, d\theta = \int_0^{2\pi} \frac{r^2}{2} \Big]_0^R d\theta = R^2 \pi$

$$\boxed{A = \pi R^2} \quad (4^*)$$

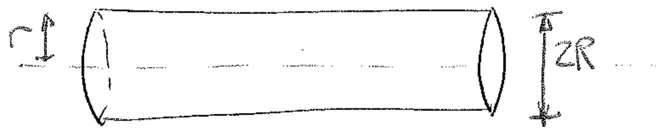
de (2), (3^*) y (4^*). $V_m = \frac{Q}{A} = \frac{V_0 \cdot \cancel{R^2} \cdot \cancel{\pi}}{3 \cdot \cancel{\pi} \cdot \cancel{R^2}}$; $\boxed{V_m = \frac{V_0}{3}}$

·) V_{eje} = ? $V_{eje} = V(r=0) = V_0 \left(1 - \frac{0}{R}\right) = V_0$ ∴

$V_m / V_{eje} = \frac{V_0}{3} : V_0 \longrightarrow \boxed{\frac{V_m}{V_{eje}} = \frac{1}{3}}$

Problema 3

Perfil de velocidades: $\boxed{u = U_0\left(1 - \frac{r^6}{R^6}\right)}$ ①



a) ¿Caudal? $\boxed{Q = u \cdot A}$ ② ; Q: caudal
A: área
u: velocidad

Para calcular, integramos la expresión ②:

$$\boxed{dQ = u \cdot dA} \quad ③$$

Tomamos elementos diferenciales de área:

$$A = \pi \cdot r^2 \longrightarrow \boxed{dA = 2\pi r\, dr} \quad ④$$

Integramos ③ con ④ y ①

$$\int_0^Q dQ = \int_0^R u \cdot 2\pi r\, dr$$

$$Q = \int_0^R U_0\left(1 - \frac{r^6}{R^6}\right) \cdot 2\pi r\, dr$$

$$Q = U_0 \cdot 2\pi \int_0^R \left(r - \frac{r^7}{R^6}\right) dr =$$

$$Q = U_0 \cdot 2\pi \left[\frac{R^2}{2} - \frac{1}{R^6} \cdot \frac{1}{8} \cdot R^8\right]$$

$$Q = U_0 \cdot 2\pi \left[\frac{R^2}{2} - \frac{R^2}{8}\right] \; ; \; Q = U_0 \cdot 2\pi \cdot R^2\left[\frac{1}{2} - \frac{1}{8}\right]$$

$$Q = U_0 \cdot 2\pi \cdot R^2 \left[\frac{4-1}{8} \right]$$

$$\boxed{Q = \frac{3}{4} \pi U_0 R^2} \quad (5)$$

verificamos unidades;

*) $\left[Q\right] = \frac{\text{Volumen}}{\text{Tiempo}} = \frac{L^3}{T}$

*) $\left[Q\right] = \left[\frac{3}{4}\pi\right] \cdot \left[U_0\right] \cdot \left[R^2\right] = 1 \cdot [\text{velocidad}] \cdot [\text{área}]$

$$\left[Q\right] = 1 \cdot \frac{L}{T} \cdot L^2 \longrightarrow \left[Q\right] = \frac{L^3}{T} \quad ✓$$

*) ¿Velocidad media (V_{med})?

$$V_{med} = \frac{\int_0^R u \cdot dr}{R} = \int_0^R \frac{U_0 (1 - r^6/R^6)}{R} dr$$

$$V_{med} = \frac{U_0}{R} \left[\left[r \right]_0^R - \frac{1}{R^6} \cdot \frac{r^7}{7} \Big]_0^R \right]$$

$$V_{med} = \frac{U_0}{R} \left[R - \frac{R^7}{7 \cdot R^6} \right] \quad ; \quad V_{med} = \frac{U_0}{R} \left[R - \frac{R}{7} \right]$$

$$V_{med} = \frac{U_0}{R} \cdot R \left[1 - \frac{1}{7} \right] \longrightarrow \boxed{V_{med} = U_0 \cdot \frac{6}{7}} \quad (6)$$

b) Esfuerzo de corte sobre paredes del tubo

Para fluidos newtonianos:

$$\boxed{\tau = \mu \cdot \frac{du}{dy}} \quad (7)$$

En este caso: $dy = dr$ (variables radio)

calculo $\frac{du}{dr}$, $\quad u = u_0(1 - r^6/R^6)$ (ec 1)

$$\frac{du}{dr} = u_0 \cdot \frac{d[1 - r^6/R^6]}{dr}$$

$$\frac{du}{dr} = u_0\left[0 - \frac{1}{R^6} \cdot 6r^5\right]$$

$$\boxed{\frac{du}{dr} = \frac{-6u_0}{R^6} \cdot r^5}, \text{ a } (7):$$

$$\boxed{\tau = \mu \cdot \left(-\frac{6u_0}{R^6}\right) \cdot r^5}; \text{ para } r = R$$

$$\tau = \frac{-\mu \cdot 6u_0}{R^6} \cdot R^5 \longrightarrow \boxed{\tau = (-6 \cdot \mu \cdot u_0)/R} \; (8)$$

verifico unidades;

$$[\tau] = \frac{\text{Fuerza}}{\text{Area}} = \frac{M \cdot L}{T^2 \cdot L^2} = \frac{M}{T^2 \cdot L}$$

$$[(-6 \cdot \mu \cdot u_0)/R] = 1 \cdot \frac{M}{L \cdot T} \cdot \frac{L}{T} \cdot L^{-1} = \frac{M}{T^2 \cdot L}$$

✓

Problema 4

a) Hallar Vmedia/Vmáx; b) hallar ÿ e Vy

a) Datos: $\boxed{u(y) = U_0 \left(\frac{y}{a}\right)^{1/7}}$ (1)

Se define $\boxed{V_{media} = \frac{Q}{A}}$ (2) ; calculo Q y A por integración. Sección transversal rectangular del conducto.

i) de (2): $Q = \int v \, dA$, $\boxed{A = l \cdot a}$ (3) — l: profundidad, a: altura

de (3): $\boxed{dA = dx\,dy}$ (4) ⟶ $\begin{cases} x \in [0, l] \\ y \in [0, a] \end{cases}$

queda: $Q = \int_0^l \int_0^a u \, dy \, dx = \int_0^l \int_0^a U_0 \left(\frac{y}{a}\right)^{\frac{1}{7}} dy \, dx \Rightarrow$

$\Rightarrow Q = a^{-1/7} U_0 \cdot l \int_0^a y^{1/7} \, dy \Rightarrow$

$\Rightarrow Q = a^{-1/7} U_0 \cdot l \cdot \frac{7}{8} y^{8/7} \Big]_0^a \Rightarrow$

$\Rightarrow Q = a^{-1/7} U_0 \cdot l \cdot \frac{7}{8} \left(a^{8/7} - 0^{8/7}\right) \Rightarrow$

$\Rightarrow \boxed{Q = a U_0 l \cdot \frac{7}{8}}$; de (3) y (2).

$V_{media} = \frac{Q}{A} = \frac{\cancel{a} U_0 \cancel{l} \cdot 7}{8 \cdot \cancel{l} \cdot \cancel{a}} \Rightarrow \boxed{V_{media} = \frac{7}{8} U_0}$ (5)

① $V_{máx} = U_0$, queda de ⑤:

$$\boxed{\frac{V_{media}}{V_{máx}} = \frac{7 \cdot \cancel{U_0}}{8 \cdot \cancel{U_0}} = \frac{7}{8}}$$

b) dado $u(y) = U_0 \left[\frac{y}{a}\right]^{\frac{1}{7}}$. ver y^* tal q' $u(y^*) = U_m$

$$\cancel{U_0}\left[\frac{y}{a}\right]^{\frac{1}{7}} = \frac{7}{8}\cdot\cancel{U_0} \Rightarrow$$

$$\Rightarrow \left(\left[\frac{y}{a}\right]^{\frac{1}{7}}\right)^7 = \left(\frac{7}{8}\right)^7 \Rightarrow \boxed{\frac{y}{a} = \left(\frac{7}{8}\right)^7 \approx 0{,}392}$$

Problema 5

El coeficiente de coriolis α (Factor de corrección de la energía) se define:

$$\boxed{\alpha = \frac{\iint v^3 \, dA}{V_{media}^3 \cdot A}} \quad (1)$$

A: Área ; V: velocidad

$dA = db \cdot dy$

.) Para la sección considerada: $\boxed{A = b \cdot h}$ (2) (rectangular)

.) $\boxed{V_{media} = \frac{V_{max}}{2}}$ (3) ; .) $\boxed{V = a \cdot y + c}$ (4) distribución lineal de velocidad

$\hookrightarrow V(0) = 0 \longrightarrow c = 0$

$\hookrightarrow V(h) = V_{max} \longrightarrow V_{max} = h \cdot a \rightarrow$

$$\boxed{V(y) = \frac{V_{max}}{h} \cdot y} \quad (5)$$

$$\boxed{dA = db \cdot dy} \quad (6)$$

queda finalmente: $\int v^3 dA = \int_0^b \int_0^h \left(\frac{V_m}{h}\right)^3 y^3 \, dy \, db = \frac{b \cdot V_m^3}{h^3} \cdot \frac{1}{4} \cdot y^4 \Big]_0^h$

$$= \frac{b \cdot V_m^3}{h^3 \cdot 4} \cdot h^4 \longrightarrow \boxed{\frac{b \cdot V_m^3 \cdot h}{4}} \quad (7)$$

En (1): $\alpha = \frac{\cancel{b} \cdot \cancel{V_m^3} \cdot \cancel{h} \cdot 2^3}{4 \cdot \cancel{V_m^3} \cdot \cancel{b} \cdot \cancel{h}} = \frac{8}{4} = 2 \longrightarrow \boxed{\boxed{\alpha = 2}}$

Problema 6

Hallar Fuerza pedida

$$\boxed{V = V_m \cdot \left[1 - \frac{4}{h^2} \cdot y^2\right]} \quad (1)$$

Para fluidos newtonianos:

$$\boxed{\tau = \mu \cdot \frac{dv}{dy}} \quad (2)$$

$$\boxed{F = \tau \cdot A} \quad (3)$$

Esquema de resolución

$$(1) \rightarrow \boxed{V} \longrightarrow (2) \longrightarrow \left.\begin{matrix}\boxed{\tau} \\ A\end{matrix}\right] \longrightarrow (3) \longrightarrow \boxed{\boxed{F}}$$

de (1) y (2):

$$\frac{dv}{dy} = \frac{d\left[V_m \cdot \left[1 - \frac{4}{h^2} \cdot y^2\right]\right]}{dy} = V_m \left(\frac{d(1)}{dy} - \frac{d\left(\frac{4}{h^2} \cdot y^2\right)}{dy}\right)$$

$$\frac{dv}{dy} = V_m \left(0 - \frac{4}{h^2} \cdot 2 \cdot y\right)$$

$$\boxed{\frac{dV}{dy} = -\frac{8 \cdot V_m \cdot y}{h^2}} \quad (4)$$

en (2):

$$\boxed{\tau = -\frac{\mu \cdot 8 \cdot V_m \cdot y}{h^2}}$$

en (3):

$$\boxed{F = -\frac{\mu \cdot 8 \cdot V_m \cdot y \cdot A}{h^2}} \quad (3)$$

En placa inferior:

$$F = -\frac{8 \cdot \mu_m \mu A \cdot y}{h_0^2}\Big]_{y=-h_0} \longrightarrow F = -\frac{8 \cdot \mu_m A \cdot (-h_0) \cdot \mu}{h_0^2}$$

$$\boxed{F = \frac{8 \cdot \mu_m \cdot A \cdot \mu}{h_0}}$$ reemplazo valores: $\mu\Big]_{15°C} = 1{,}139 \cdot 10^{-3} \frac{kg}{m \cdot s}$

$h_0 = 0{,}5\,mm$

$$F = \frac{8 \cdot 0{,}05\,\cancel{m} \cdot 0{,}1\,m^2 \cdot \cancel{10^3}\,\cancel{mm} \cdot \cancel{10^{-3}}\,kg \cdot 1{,}139}{1 \cdot 0{,}5\,\cancel{mm} \cdot 1\,\cancel{m} \quad \cancel{m} \cdot s}$$

$$[F] = m \cdot \frac{kg}{s^2} = N \qquad \boxed{F = 2{,}278 \cdot 10^{-2}\,N}$$

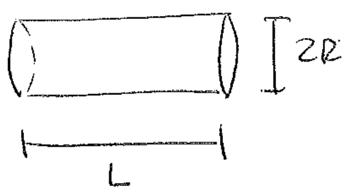
Problema 7

Hallar F

Ecuaciones

$$\boxed{F = \tau \cdot A} \quad (1)$$

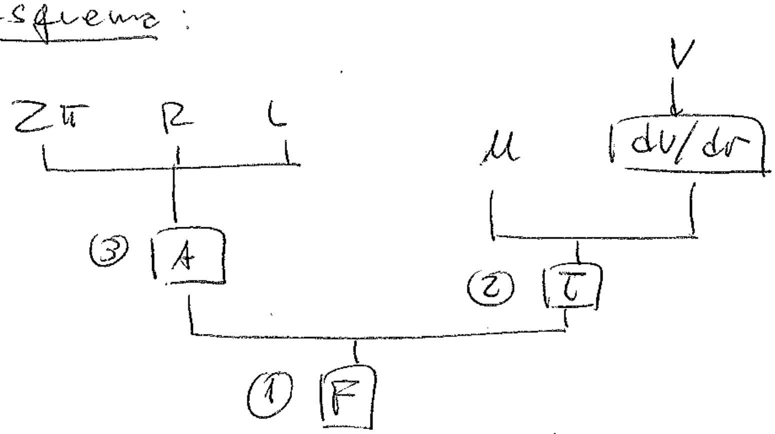
$$\boxed{\tau = \mu \cdot \frac{dv}{dr}} \quad (2)$$



$\rightarrow \boxed{A = 2\pi R \cdot L}$ (3)

Area por unidad de longitud "L"

Esquema:



*) $$\frac{dv}{dr} = \frac{d\left(V_m\left(1 - r^n/R^n\right)\right)}{dr} = V_m\left[\frac{d(1)}{dr} - \frac{1}{R^n}\cdot\frac{d(r^n)}{dr}\right]$$

$$\frac{dv}{dr} = V_m\left[0 - \frac{1}{R^n}\cdot n \cdot r^{n-1}\right] \; ; \quad \boxed{\frac{dv}{dr} = \frac{-n \cdot r^{n-1} \cdot V_m}{R^n}}$$

evaluada en $r = R$

$$\boxed{\frac{dv}{dr} = \frac{-n \cdot R^{n-1} \cdot V_m}{R^n} = \frac{-n \cdot V_m}{R}}$$

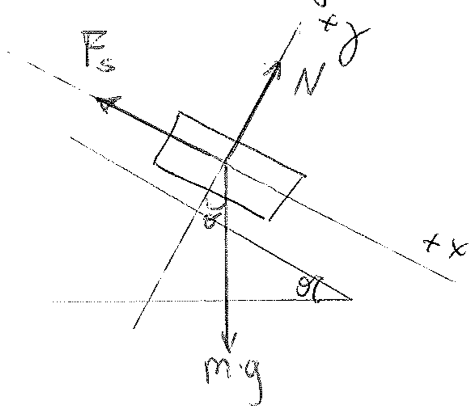
reemplazo en (1):

$$\boxed{\boxed{F = 2\pi R L \cdot \mu \cdot \left(-\frac{n \cdot V_m}{R}\right)}}$$

Problema 8

Hallar velocidad final U_B

(*) Planteo diagrama de cuerpo libre



donde F_s: Fuerza de fricción fluido

$m \cdot g$: peso del bloque

Dado que U_B = constante, ∴

$\boxed{\sum F_x = 0}$ (1) $\boxed{\sum F_y = 0}$ (2)

Para calcular F_s:

$$\tau = \frac{F_s}{\text{Area}} = \frac{F_s}{S} \longrightarrow \boxed{F_s = \tau \cdot S} \quad (3)$$

Para fluidos newtonianos:

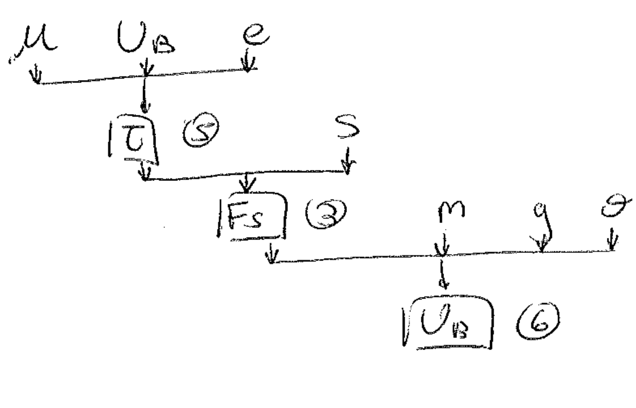
$$\boxed{\tau = \mu \cdot \frac{dV}{dy}} \quad (4) \longrightarrow \text{en perfil lineal: } \boxed{\tau = \mu \cdot \frac{U_B}{e}} \quad (5)$$

De ec. (1):

$$\sum F_x = 0$$

$$\boxed{F_s - m \cdot g \cdot \sin(\theta) = 0} \quad (6)$$

*) Esquema de resolución



de 5 y 3:

$$\boxed{F_S = \frac{\mu \cdot V_B \cdot S}{e}} \quad (3^*)$$

de 3^* y 6:

$$F_S - m \cdot g \cdot \operatorname{sen}(\theta) = 0$$

$$\frac{\mu \cdot V_B \cdot S}{e} - m \cdot g \cdot \operatorname{sen}(\theta) = 0$$

$$\frac{\mu \cdot V_B \cdot S}{e} = m \cdot g \cdot \operatorname{sen}(\theta)$$

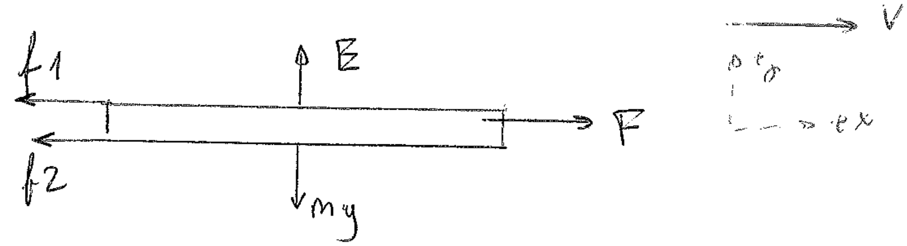
$$\frac{\cancel{\mu} \cdot V_B \cdot \cancel{S}}{\cancel{e}} \cdot \frac{\cancel{e}}{\cancel{S \cdot \mu}} = m \cdot g \cdot \operatorname{sen}(\theta) \cdot \frac{e}{S \cdot \mu}$$

$$\boxed{V_B = \frac{m \cdot g \cdot \operatorname{sen}(\theta) \cdot e}{S \cdot \mu}}$$

Problema 9

Hallar fuerza F

* Planteamos diagrama de cuerpo libre de la placa móvil



Donde: f_1, f_2: fricción fluido de fluido 1 y 2

E: empuje

$m \cdot g$ = peso

F: fuerza aplicada a placa.

*) Para velocidad constante "V": $\boxed{\sum F_x = 0}$ (1) $\boxed{\sum F_y = 0}$ (2)

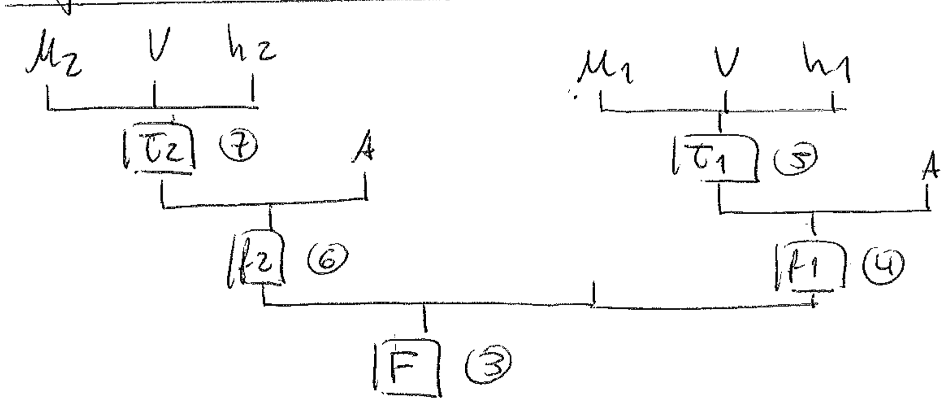
$$\sum F = F - f_1 - f_2 = 0$$

$$\boxed{F = f_1 + f_2} \quad (3)$$

* $\boxed{\tau_1 = \frac{f_1}{A}}$ (4) ; con fluidos newtonianos; $\boxed{\tau_1 = \frac{\mu_1 \cdot V}{h_1}}$ (5)

* $\boxed{\tau_2 = \frac{f_2}{A}}$ (6) , $\boxed{\tau_2 = \frac{\mu_2 \cdot V}{h_2}}$ (7)

Esquema de resolución



de 7 en 6 y 5 y 4

$$\boxed{f_1 = \frac{\mu_1 \cdot V}{h_1} \cdot A} \qquad \boxed{f_2 = \frac{\mu_2 \cdot V \cdot A}{h_2}}$$

de 3:

$$F = \frac{\mu_1 \cdot V \cdot A}{h_1} + \frac{\mu_2 \cdot V \cdot A}{h_2}$$

$$\boxed{F = V \cdot A \left(\frac{\mu_1}{h_1} + \frac{\mu_2}{h_2} \right)} \quad (8)$$

Verificamos unidades:

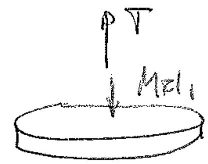
$$[F] = \frac{M \cdot L}{T^2}$$

$$[F] = [V] \cdot [A] \cdot \left[\frac{\mu}{h}\right] = \frac{L}{T} \cdot \cancel{L^2} \cdot \frac{M}{\cancel{L} \cdot T} \cdot \frac{1}{\cancel{L}} \quad \therefore [F] = \frac{M \cdot L}{T^2} \quad \checkmark$$

Problema 10

Hallar ω_2

para el disco 1, $\omega_1 = cte$, $\therefore$ $\boxed{\Sigma M = 0}$ ①

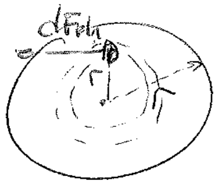


M_{fl1} = momento aplicado por fricción fluido en disco 1.

de ①: $\boxed{T - M_{fl1} = 0}$ ②

$M_{fl1} = F_{fl1} \cdot R$ → resuelvo con integral:

$\boxed{dM_{fl1} = r \cdot dF_{fl1}}$ ③



$\boxed{dF_{fl1} = \tau \cdot dA}$ ④

Tomo áreas de anillo:

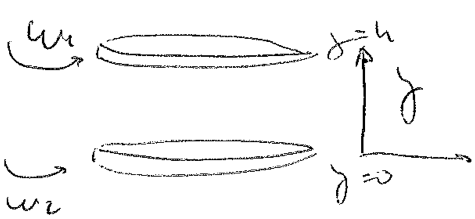
$A = \pi \cdot r^2 \longrightarrow \boxed{dA = 2\pi r\, dr}$ ⑤

Supongamos fluido newtoniano,

$\boxed{\tau = \mu \cdot \frac{dv}{dy}}$ ⑥ Siendo y: coordenada vertical ~~normal~~ a disco 1 y 2.

Para perfil lineal, $\boxed{V = a \cdot y + b}$ ⑦, $a, b \in \mathbb{R}$

El fluido tiene la misma velocidad ~~que~~ los puntos de contacto con los discos que los propios discos.



$$\left.\begin{aligned} V(0) &= \omega_2 \cdot r \\ V(h) &= \omega_1 \cdot r \end{aligned}\right\}$$

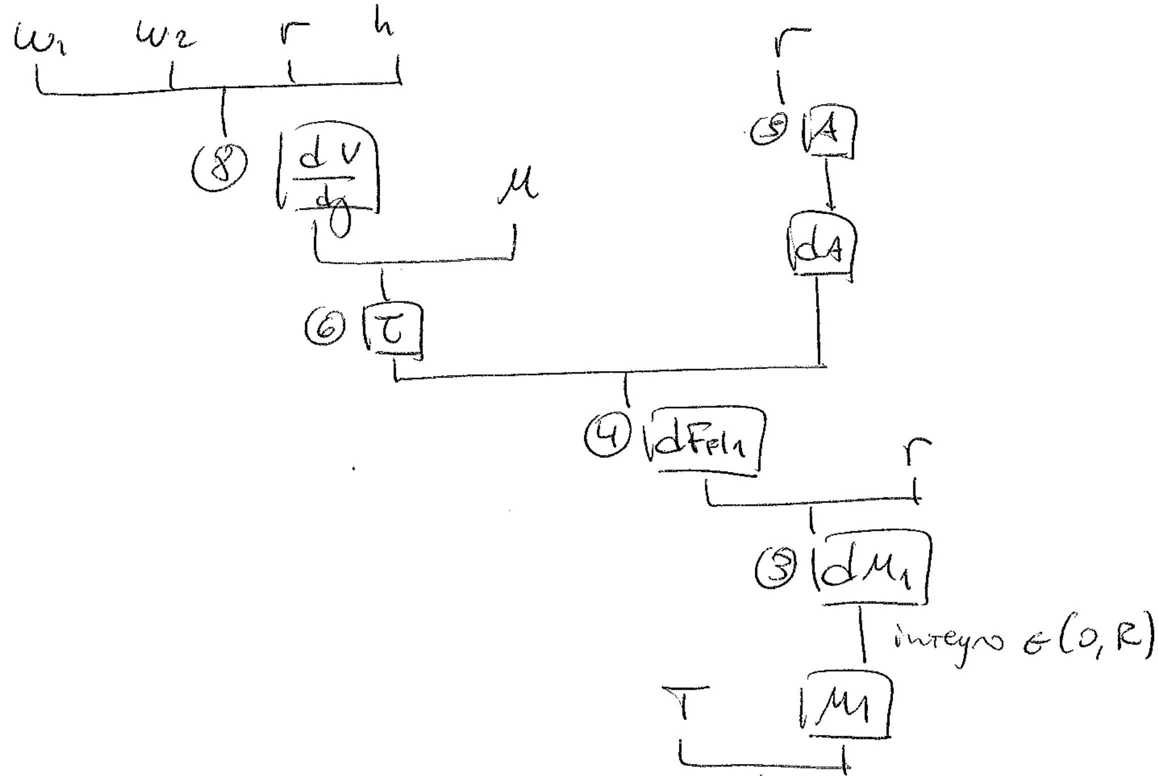
reemplazo p/ calcular "a" y "b"

$\boxed{b = \omega_2 \cdot r}$ $\boxed{a = (\omega_1 - \omega_2) \cdot \frac{r}{h}}$

$$\boxed{V = \frac{r}{h}(\omega_1 - \omega_2) \cdot y + \omega_2 \cdot r}$$ ⑦

$$\boxed{\frac{dv}{dy} = \frac{r}{h}(\omega_1 - \omega_2)} \quad ⑧$$

<u>Esquema de resolución</u>



② ω_2

de ⑧ y ⑥: $\tau = \mu \cdot \frac{r}{h}(\omega_1 - \omega_2)$

de ⑥ y ⑤ en ④: $dF_{Fl1} = \frac{\mu \cdot r}{h} \cdot (\omega_1 - \omega_2) \cdot 2\pi r\, dr$

de ③: $\int_0^R dM_1 = \int_0^R \frac{\mu}{h} \cdot 2\pi (\omega_1 - \omega_2) \cdot r^3\, dr$

$$M_1 = \frac{\mu \cdot 2\pi (\omega_1 - \omega_2) \cdot R^4}{4h}, \quad \text{con } R = \phi/2$$

$$M_1 = \frac{\mu \cdot 2\pi (\omega_1 - \omega_2) \cdot (\phi/2)^4}{4h} = T$$

de ②:

$$\omega_1 - \omega_2 = \frac{T \cdot 4 \cdot h}{2\pi \mu (\phi/2)^4}$$

$$\boxed{\omega_2 = \omega_1 - \frac{T \cdot 4 \cdot h}{2\pi \mu (\phi/2)^4}}$$