Facultad de Ciencias Fisicomatemáticas
e Ingeniería

MECÁNICA DE LOS FLUIDOS

EJERCICIOS ADICIONALES

CLASES TEÓRICAS

CINEMÁTICA DE FLUIDOS

UNIVERSIDAD CATÓLICA ARGENTINA - *SANTA MARÍA DE LOS BUENOS AIRES*
FACULTAD DE FISICOMATEMÁTICAS E INGENIERÍA
CÁTEDRA DE MECÁNICA DE FLUIDOS

GUIA TEÓRICA 3: CINEMÁTICA DE FLUIDOS

Problema 1

La velocidad de un flujo tridimensional está dada por:

$$\mathbf{V} = yz\hat{\mathbf{i}} + x^2z\hat{\mathbf{j}} + x\hat{\mathbf{k}}$$

Determinar las componentes rectangulares de la aceleración.

④ Hallar componentes rectangulares de la aceleración; con:

$$\boxed{\bar{V} = (yz;\ x^2 z;\ x)} \quad (1)$$

Según nomenclatura:

$$\bar{V} = u(x,y)\,\hat{i} + v(x,y)\,\hat{j} + w(x,y)\,\hat{k}$$

Por ecuación Euler-Lagrange:

$$\boxed{\bar{a} = \frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + \bar{V}\cdot(\bar{\nabla}\cdot\bar{V})} \quad (2)$$

En componentes:

$$(3)\quad \bar{a} = \begin{cases} a_x = \dfrac{\partial u}{\partial t} + u\cdot\dfrac{\partial u}{\partial x} + v\cdot\dfrac{\partial u}{\partial y} + w\cdot\dfrac{\partial u}{\partial z} \\[2ex] a_y = \dfrac{\partial v}{\partial t} + u\cdot\dfrac{\partial v}{\partial x} + v\cdot\dfrac{\partial v}{\partial y} + w\cdot\dfrac{\partial v}{\partial z} \\[2ex] a_z = \dfrac{\partial w}{\partial t} + u\cdot\dfrac{\partial w}{\partial x} + v\cdot\dfrac{\partial w}{\partial y} + w\cdot\dfrac{\partial u}{\partial z} \end{cases}$$

Por ecuación 1 y 3

$a_x = 0 + yz \cdot 0 + x^2 z \cdot z + x \cdot y$

$\boxed{a_x = x^2 z^2 + xy}$

$a_y = 0 + yz \cdot 2xz + x^2 z \cdot 0 + x \cdot x^2$

$\boxed{a_y = 2xyz^2 + x^3}$

$a_z = 0 + yz \cdot 1 + x^2 z \cdot 0 + x \cdot 0$

$\boxed{a_z = yz}$, Queda:

$$\boxed{\bar{a} = [(xz)^2 + xy]\,\hat{i} + [2xyz^2 + x^3]\,\hat{j} + [yz]\,\hat{k}} \quad (4)$$

Problema 2

Determinar la función corriente correspondiente al siguiente potencial:

$$\phi = x^3 - 3xy^2$$

Dado $\boxed{\phi = x^3 - 3xy^2}$ (1)

Para función corriente (ψ), dado $\bar{V}: \mathbb{R}^2 \to \mathbb{R}^2$

$\bar{V} = (u, v)$; $u = f(x,y)$; $v = g(x,y)$

se cumple:

$$\boxed{u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}} \ (2) \quad ; \quad \boxed{v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}} \ (3)$$

con ecuaciones (2), (3) y (1); derivando e integrando, buscamos ψ:

$$\textcircled{*}\begin{cases} \dfrac{d\phi}{dx} = \dfrac{d(x^3-3xy^2)}{dx} = 3x^2-3y^2 \\ \dfrac{d\psi}{dy} = \dfrac{d\phi}{dx} \end{cases}$$

$$\therefore \frac{d\psi}{dy} = 3x^2 - 3y^2$$

$$\int_0^{\psi} d\psi = \int_0^{y} (3x^2-3y^2)\cdot dy$$

$$\boxed{\psi = 3x^2y - y^3 + f(x)} \quad (4)$$

$$\textcircled{*}\ \frac{\partial\phi}{\partial y} = 0 - 3\cdot 2\cdot x\cdot y \qquad -6xy = -6xy + f'(x)$$

$$\text{de } (4): -\frac{\partial\psi}{\partial x} = -6xy - f'(x) \qquad f'(x) = 0$$

$$\frac{df}{dx} = 0 \longrightarrow f = cte\ (C) \text{; queda:}$$

$$\boxed{\psi = 3x^2y - y^3 + C} \quad (5)$$

Problema 3

Las componentes de velocidad de un campo de velocidades están dadas por:

$U = x^2y \quad V = -xy^2$

a. hallar la ecuación para las líneas de corriente y graficar la que pasa por el punto (1,1).
b. Hallar la función corriente.

Problema 3

Dado el campo de velocidades $\bar{V}: \mathbb{R}^2 \to \mathbb{R}^2$

$$\bar{V} = (x^2 y ; -x y^2) :$$

a) ¿Línea de corriente por $(1,1)$?

Dado el vector posición $\bar{r}: t \in \mathbb{R} \to \mathbb{R}^2$; se define $\bar{V}$ como:

$$\bar{V} = \frac{d\bar{r}}{dt} = \left(\frac{dx}{dt} ; \frac{dy}{dt}\right) = (u;v)$$

, de acá despejo:

$$\left.\begin{array}{ll} u = \dfrac{dx}{dt} ; \Rightarrow & dt = \dfrac{dx}{u} \\ v = \dfrac{dy}{dt} ; \Rightarrow & dt = \dfrac{dy}{v} \end{array}\right\} \Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v}} \quad (1)$$

Integro ec. (1):

$$\frac{dx}{u} = \frac{dx}{x^2 y} \quad ; \quad \frac{dy}{v} = \frac{dy}{-x y^2} \quad ; \quad \therefore$$

$$\frac{dx}{\cancel{x^2 y}} = \frac{dy}{\cancel{-x y^2}} \Rightarrow \frac{dx}{x} = -\frac{dy}{y} \Rightarrow$$

$$\Rightarrow \int \frac{dx}{x} = (-1) \cdot \int \frac{dy}{y} \Rightarrow (-1) \cdot \ln|x| = \ln|y| + \ln|c|$$

donde $c \in \mathbb{R}$,

$$e^{[(-1) \cdot \ln|x|]} = e^{[\ln|y| + \ln|c|]}$$

$$|x|^{-1} = |y| \cdot |c| \Rightarrow \boxed{|y| = \frac{|c|}{|x|}} \quad (2)$$

Para el punto (1,1), de la familia de curvas descriptas en ②, se cumple:

$\boxed{y = \frac{C}{x}}$ → primer cuadrante, $x \geq 0$, $y \geq 0$
busco valor de C para $x=1$, $y=1$

$1 = \frac{C}{1} \rightarrow C = 1$; $\therefore$ $\boxed{y = \frac{1}{x}}$ ③

③: línea de corriente por (1,1)

b) {Verifico función corriente!}

$\boxed{\psi = \frac{1}{2}x^2y^2 + C}$

$u = \frac{\partial \psi}{\partial y} = \frac{\partial\left(\frac{1}{2}x^2y^2 + C\right)}{\partial y} = \frac{1}{2}x^2 \cdot 2y + 0 = x^2y$

$u = x^2y$ ✓

$v = -\frac{\partial \psi}{\partial x} = (-1) \cdot \frac{\partial\left(\frac{1}{2}x^2y^2 + C\right)}{\partial x} = -x \cdot y^2$

$v = -xy^2$ ✓

Problema 4

El movimiento de un fluido incompresible se realiza bajo la acción de un campo de velocidades:

$$\vec{v} = xt\,\vec{i} + 2yt\,\vec{j} + zt\,\vec{k}$$

y un campo de fuerzas:

$$F = x\,\vec{i} + 2y\,\vec{j} + z\,\vec{k}$$

Determinar:

a. La familia de líneas de corriente y las trayectorias de las partículas, indicando si coinciden o no.
b. Campo de presiones.

a) 1) Trayectorias de partículas:

$$\bar{V} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \quad ; \quad \bar{V} = (xt; 2yt; zt)$$

Tenemos: ① $\frac{dx}{dt} = xt$

$$\int \frac{dx}{x} = \int t\, dt \Rightarrow \ln|x| = \frac{t^2}{2} + c_1 \Rightarrow$$

$$e^{\ln|x|} = e^{(t^2/2 + c_1)}$$

$$e^{c_1} = K_1$$

$$x(t) = K_1 * e^{\frac{t^2}{2}} \quad (1)$$

$$\frac{dy}{dt} = 2yt \Rightarrow \frac{1}{2}\int \frac{dy}{y} = \int t\, dt \Rightarrow$$

$$\Rightarrow \cancel{\frac{1}{2}} \ln|y| = \cancel{\frac{1}{2}} t^2 + c_2 \Rightarrow$$

$$e^{c_2} = K_2$$

$$y(t) = K_2 * e^{t^2} \quad (2)$$

$$\frac{dz}{dt} = zt \Rightarrow \int \frac{dz}{z} = \int t\, dt \Rightarrow \ln|z| = \frac{t^2}{2} + c_3 \Rightarrow$$

$$\Rightarrow e^{\ln|z|} = e^{(t^2/2 + c_3)}$$

$$e^{c_3} = K_3$$

$$z(t) = K_3 * e^{\frac{t^2}{2}} \quad (3)$$

Vemos que; dado el vector posición $\bar{r}(t) : t \in \mathbb{R} \to \mathbb{R}^3$

$$\bar{r}(t) = (x(t);\ y(t);\ z(t))$$

$$\overline{r(t)} = \left(K_1 * e^{\frac{t^2}{2}}; K_2 * e^{t^2}; K_3 * e^{\frac{t^2}{2}}\right) \quad (4)$$

2) Lineas de corriente:

$$\boxed{\frac{dx}{\mu} = \frac{dy}{v} = \frac{dz}{w}} \quad (5)$$

Tenemos: $\frac{dx}{x\cancel{t}} = \frac{dy}{2y\cancel{t}} \Rightarrow \int \frac{dx}{x} = \frac{1}{2}\int \frac{dy}{y} \Rightarrow$

$$\Rightarrow e^{\ln|x|} = e^{\left[\frac{1}{2}\ln|y| + \ln|C_4|\right]} \Rightarrow$$

$$\boxed{|x| = |y|^{\frac{1}{2}} |C_4|} \quad (6)$$

) $\frac{dy}{v} = \frac{dz}{w} \Rightarrow \frac{dy}{2y\cancel{t}} = \frac{dz}{2z\cancel{t}} \Rightarrow \frac{1}{2}\int \frac{dy}{y} = \int \frac{dz}{z} \Rightarrow$

$$\Rightarrow e^{\frac{1}{2}\ln|y|} = e^{(\ln|z| + \ln|C_5|)} \Rightarrow \boxed{|y|^{\frac{1}{2}} = |z|\,|C_5|} \quad (7)$$

) $\frac{dx}{\mu} = \frac{dz}{\mu} \Rightarrow \frac{dx}{x\cancel{t}} = \frac{dz}{z\cancel{t}} \Rightarrow \int \frac{dx}{x} = \int \frac{dz}{z} \Rightarrow$

$$\Rightarrow e^{\ln|x|} = e^{(\ln|z| + \ln|C_6|)} \Rightarrow \boxed{|x| = |z| \cdot |C_6|} \quad (8)$$

b) ¿Campo de presiones? dada la fórmula del campo gradiente de presiones (citado del White)

$$\boxed{\bar{\nabla} p = \rho \cdot (\bar{g} - \bar{a}) + \mu \cdot \nabla^2 \cdot \bar{V}} \quad (9)$$

donde

$$\boxed{\bar{a} = \frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot (\bar{\nabla} \cdot \bar{V})} \quad (10)$$

·) De 10 $\left[\frac{\partial \bar{V}}{\partial t} = \left(\frac{\partial (xt)}{\partial t}; \frac{\partial (2yt)}{\partial t}, \frac{\partial (zt)}{\partial t} \right) = (x, 2y, z) \right]$

·) $\frac{\partial u}{\partial x} = \frac{\partial (xt)}{\partial x} = t$, ·) $\frac{\partial u}{\partial y} = \frac{\partial (xt)}{\partial y} = 0$; ·) $\frac{\partial u}{\partial z} = 0$

·) $\frac{\partial v}{\partial x} = \frac{\partial (2yt)}{\partial x} = 0$; ·) $\frac{\partial v}{\partial y} = \frac{\partial (2yt)}{\partial y} = 2t$; ·) $\frac{\partial v}{\partial z} = 0$

·) $\frac{\partial w}{\partial x} = \frac{\partial (zt)}{\partial x} = 0$; ·) $\frac{\partial w}{\partial y} = \frac{\partial (zt)}{\partial y} = 0$; ·) $\frac{\partial w}{\partial z} = \frac{\partial (zt)}{\partial z} = t$

queda: ·) $a_x = \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}$

$a_x = x + x t \cdot t + v \cdot 0 + w \cdot 0$, $\boxed{a_x = x + x t^2}$ (11)

·) $a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$

$a_y = 2y + u\cdot 0 + 2yt\cdot 2t + w\cdot 0$; $\boxed{a_y = 2y(1+2t^2)}$ (12)

·) $a_z = \frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$

$a_z = z + u\cdot 0 + v\cdot 0 + z t t$; $\boxed{a_y = z + zt^2}$ (13)

queda: $\boxed{\bar{a} = (x + xt^2;\ 2y(1+2t^2);\ z + zt^2)}$ (14)

·) De 10: $\boxed{\mu\bar{\nabla}^2\cdot\bar{V} = \mu\left(\frac{\partial^2\bar{V}}{\partial x^2} + \frac{\partial^2\bar{V}}{\partial y^2} + \frac{\partial^2\bar{V}}{\partial z^2}\right)}$ (15)

de 15:

$$\begin{cases} \frac{\partial^2\bar{V}}{\partial x^2} = \left(\frac{\partial^2 u}{\partial x^2};\ \frac{\partial^2 v}{\partial x^2},\ \frac{\partial^2 w}{\partial x^2}\right) = \left(\frac{\partial(t)}{\partial x};\ \frac{\partial(0)}{\partial x},\ \frac{\partial(0)}{\partial x}\right) = (\bar{0}) \\ \frac{\partial^2\bar{V}}{\partial y^2} = \left(\frac{\partial^2 u}{\partial y^2},\ \frac{\partial^2 v}{\partial y^2},\ \frac{\partial^2 w}{\partial y^2}\right) = \left(0,\ \frac{\partial(2t)}{\partial y},\ 0\right) = (\bar{0}) \\ \frac{\partial^2\bar{V}}{\partial z^2} = \left(\frac{\partial^2 u}{\partial z^2};\ \frac{\partial^2 v}{\partial z^2};\ \frac{\partial^2 w}{\partial z^2}\right) = \bar{0} \end{cases}$$

$\therefore$ $\boxed{\mu\nabla^2 V = (0,0,0)}$ (15*) , de (15*) y (14) en (9):

$\bar{\nabla}p = \delta\left((0,g,0) - (a_x, a_y, a_z)\right)$

$\left(\frac{\partial p}{\partial x};\ \frac{\partial p}{\partial y};\ \frac{\partial p}{\partial z}\right) = \delta(-a_x,\ g - a_y,\ -a_z)$. integro por pasos:

·) $\frac{\partial p}{\partial x} = -\delta\cdot a_x \Rightarrow \int \partial p = -\int \delta\cdot a_x\cdot\partial x$

$p = -\delta\int a_x\, dx + f(y,z)$; reemplazo a_x

$$P = (-\delta)\cdot\int (x + x\pi^2)\,dx + g(y,z)$$

$$P = (-\delta)\left[\frac{x^2}{2} + \frac{x^2}{2}\pi^2\right] + g(y,z)$$

$$\boxed{P = -\frac{\delta x^2}{2}(1+\pi^2) + g(y,z)} \quad (16)$$

(8) $$\frac{\partial P}{\partial y} = \delta\cdot(y - a_y) \Rightarrow \int \partial P = \int \delta\cdot(y - a_y)\,\partial y \Rightarrow$$

$$\Rightarrow P = \delta\left[\int g\,dy - \int a_y\,dy\right] + h(z)$$

$$P = \delta\cdot g\cdot y - \delta\int zy(1+2\pi^2)\,dy + h(z)$$

$$\boxed{P = \delta\cdot g\, y - \delta(1+2\pi^2)\,y^2 + h(x,z)} \quad (17)$$

de (16): $$\frac{\partial P}{\partial y} = 0 + g_y \; ; \; \frac{\partial P}{\partial y} = \delta\cdot(g - a_y)$$

$$g_y = \delta g - \delta\cdot a_y$$

$$\frac{\partial g}{\partial y} = \delta g - \delta\left(zy(1+2\pi^2)\right)$$

$$\int \partial g = \int\left[\delta g - \delta zy(1+2\pi^2)\right]\partial y$$

$$\boxed{g = \delta\cdot g\cdot y - \delta y^2(1+2\pi^2) + m(z)} \quad (18)$$

de (18) y (16): (19) $$\boxed{P = -\frac{\delta x^2}{2}(1+\pi^2) + \delta g y - \delta\cdot y^2(1+2\pi^2) + m(z)}$$

de (9): se cumple para (17).

) $$\frac{\partial P}{\partial z} = -\delta\cdot a z \Rightarrow \frac{\partial P}{\partial z} = -\delta\cdot(z + z\pi^2)$$

de (19): $$\frac{\partial P}{\partial z} = 0 + 0 + 0 + \frac{\partial m}{\partial z}$$

$$\frac{dm}{dz} = -\delta z(1+\tau^2) \Rightarrow \int dm = \int -\delta(1+\tau^2) z \, dz \Rightarrow \quad (19)$$

$$\Rightarrow \boxed{m(z) = -\frac{\delta}{2}(1+\tau^2) z^2 + C} \quad (20)$$

de (20) y (19):

$$\boxed{P(x,y,z,\tau) = -\frac{\delta}{2} x^2(1+\tau^2) + \delta g y - \delta y^2(1+2\tau^2) - \frac{\delta}{2}(1+\tau^2) z^2 + C}$$

Problema 5

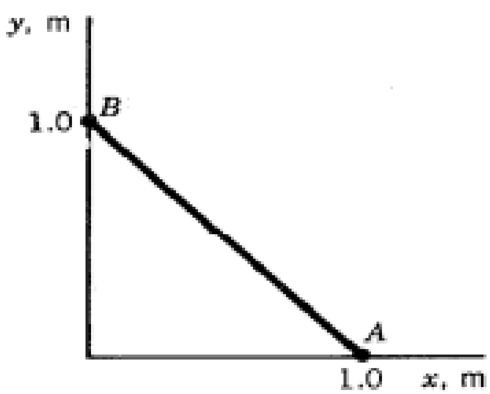
La función corriente de un flujo incompresible está dada por la siguiente expresión:

$$\Psi = 3x^2 y - y^3$$

Con unidades de m^2/s, estando x e y en metros.

c. Dibujar las líneas de corriente a través del origen.

d. Determinar el caudal a través de los puntos A y B mostrados en la figura.



$$\boxed{\psi = 3x^2 y - y^3} \quad (1)$$

Para líneas de corriente, ψ = cte.

Dibujamos con $\psi = -2, -1, 0, 1, 2$

*) $\psi_0 = 0 = 3x^2 y - y^3 \longrightarrow 3x^2 - y^2 = 0 \rightarrow y = \pm\sqrt{3}\cdot|x|$

$\psi_0 : y = \pm\sqrt{3} \cdot x$

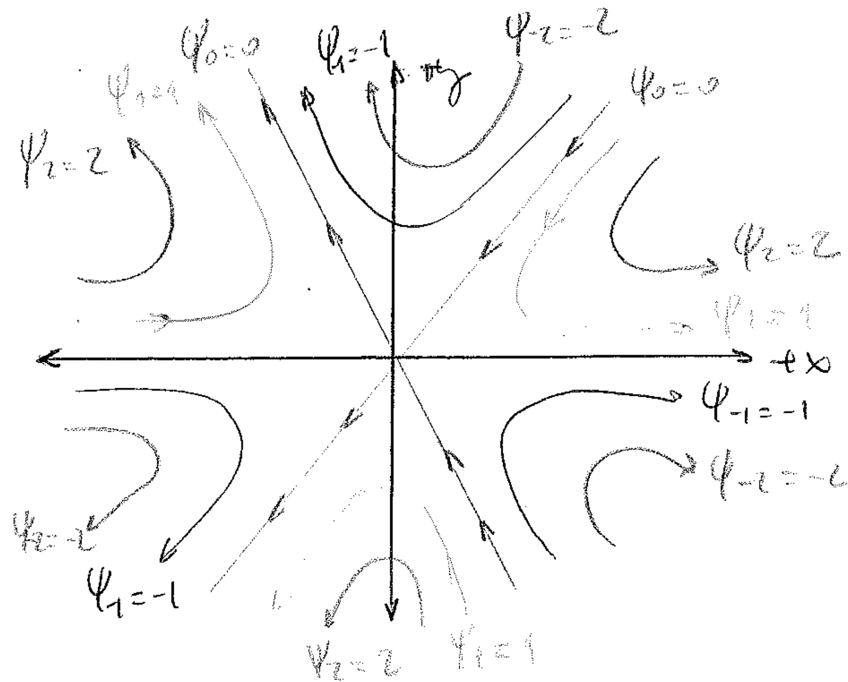
*) $\psi_1 = 1 = 3x^2 y - y^3 \Rightarrow (3x^2 - y^2)\cdot y = 1$

*) $\psi_1 = -1 = 3x^2 - y^3 \Rightarrow (y^2 - 3x^2)\cdot y = 1$

*) $\psi_{-2} = -2 = 3x^2 - y^3 \longrightarrow (y^2 - 3x^2)\cdot y = 2$

*) $\psi_2 = 2 = 3x^2 - y^3 \longrightarrow (y^2 - 3x^2)\cdot(-y) = 2$

Tip: Esquema grafico similar al ejemplo 4.7, capitulo 7, White:



b) Calculo caudal entre A y B

Segun White: $\boxed{Q = \int_1^2 d\psi = \psi_2 - \psi_1}$ ①

donde se toma ancho unitario.

queda: $\psi_2 = \psi(1,0) = 3\cdot 1^2 \cdot 0 - 0^3 = 0$

$\psi_1 = \psi(0,1) = 3\cdot 0^2 \cdot 1 - 1^3 = -1$

$Q = [0 - -1] \frac{m^2}{s} \cdot m \rightarrow \boxed{Q = 1\ m^3/s}$

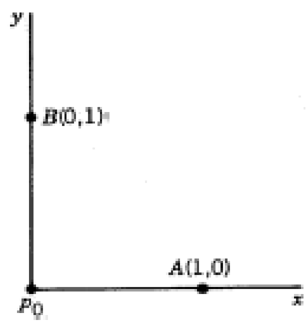
Problema 6

Un flujo bidimensional, no viscoso, incompresible, tiene un campo cuyas componentes son:

$$U = U_0 + 2y$$

$V = 0$

donde U_0 es constante. Si la presión en el origen es p_0, determinar la expresión para la presión en el punto A y en el punto B. Justificar la respuesta. Asumir que las unidades son consistentes y que las fuerzas sobre el cuerpo pueden ser despreciadas.



Problema 6 Dado $\bar{V}: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

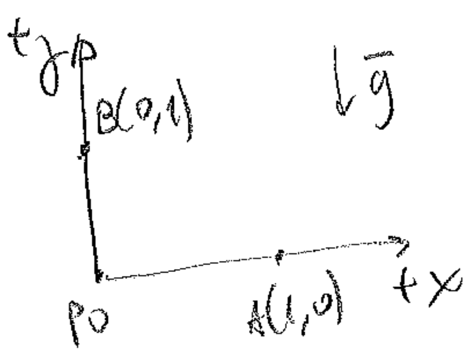
$\boxed{\bar{V}(x,y) = (U_0 + 2y, 0)}$ ① ; $U_0 \in \mathbb{R}$. presión en origen p_0

Hallar Δp_{AB}

Se define el campo gradiente de presiones como:

$$\boxed{\bar{\nabla} p = \delta(\bar{g} - \bar{a}) + \mu \nabla^2 \bar{V}} \quad ②$$

de ②: buscar $\bar{a}$, $\bar{g}$, $\bar{V}$; integrar en pasos:



$\downarrow \bar{g}$

$$\boxed{\bar{g} = -g\hat{j}} \quad ③$$

③ Calculemos $\bar{a}$ con ecuación Euler-Lagrange / derivada total:

$$\boxed{\bar{a} = \frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + \bar{V}\cdot(\bar{\nabla}\cdot\bar{V})}\ (4)$$

$\begin{cases} \text{con } w=0 \\ \text{con } v=0 \end{cases}$ ∴

$$\bar{a} = \begin{cases} a_x = \dfrac{\partial u}{\partial t} + u\cdot\dfrac{\partial u}{\partial x} + \cancel{v\cdot\dfrac{\partial u}{\partial y}} + \cancel{w\cdot\dfrac{\partial u}{\partial z}} \\ a_y = \cancel{\dfrac{\partial v}{\partial t}} + \cancel{u\cdot\dfrac{\partial v}{\partial x}} + \cancel{v\cdot\dfrac{\partial v}{\partial y}} + \cancel{w\cdot\dfrac{\partial v}{\partial z}} \\ a_z = \cancel{\dfrac{\partial w}{\partial t}} + \cancel{u\cdot\dfrac{\partial w}{\partial x}} + \cancel{v\cdot\dfrac{\partial w}{\partial y}} + \cancel{w\cdot\dfrac{\partial w}{\partial z}} \end{cases}$$

$$\therefore\ \boxed{\bar{a} = \left(\frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x}\right)\hat{i}}\ (4^*)$$

Calculo componentes de (4*):

$$\left.\begin{array}{l} \cdot)\ \dfrac{\partial u}{\partial t} = \dfrac{\partial(U_0 + 2y)}{\partial t} = 0 \\ \cdot)\ \dfrac{\partial u}{\partial x} = \dfrac{\partial(U_0 + 2y)}{\partial x} = 0 \end{array}\right\} \Rightarrow \bar{a} = (0 + u\cdot 0)\hat{i}$$

$$\therefore\ \boxed{\bar{a} = \bar{0}}\ (4^{**})$$

④ Calculo $\mu\nabla^2\bar{V}$:

$$\bar{\nabla}\cdot\bar{V} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(u, v, w) = \left(\cancel{\frac{\partial u}{\partial x}}^{\,0} + \cancel{\frac{\partial v}{\partial y}}^{\,0} + \cancel{\frac{\partial w}{\partial z}}^{\,0}\right)$$

$$\bar{\nabla}\cdot\bar{V} = 0 \Rightarrow \boxed{\mu\nabla^2 V = 0}\ (5)$$

④ de (5), (4**), ③ y ②

$$\bar{\nabla} p = \delta \cdot (\bar{y} - \cancel{\bar{a}}^{\,0}) + \cancel{\mu \bar{\nabla}^2 \bar{v}}^{\,0}$$

$$\bar{\nabla} p = \delta(\bar{y}) \Rightarrow \bar{\nabla} p = -\delta g \hat{j} \Rightarrow$$

$$\Rightarrow \left(\frac{\partial p}{\partial x} ; \frac{\partial p}{\partial y} \right) = -\delta \cdot g \cdot \hat{j}$$

$\cdot)$ $\frac{\partial p}{\partial x} = 0 \longrightarrow \boxed{p = C_1 + h(y)}$ (6)

$\cdot)$ $\frac{\partial p}{\partial y} = -\delta \cdot g$; $\frac{\partial p}{\partial y} = \frac{\partial h}{\partial y} \Rightarrow -\delta g = \frac{\partial h}{\partial y} \Rightarrow$

$$\Rightarrow dh = -\delta g \, \partial y \Rightarrow \boxed{h = -\delta g y + C_2} \quad (7)$$

de (6) y (7) queda: $\boxed{p = C_1 - \delta \cdot g \cdot y + C_2}$ (6*)

$$\boxed{p(x,y) = -\delta \cdot g y + C} \quad (6^*), \quad C \in \mathbb{R}$$

Se sabe: $p(0,0) = p_0$

$$p(0,0) = -\delta \cdot g \cdot 0 + C = p_0 \longrightarrow C = p_0;$$

$$\boxed{p(x,y) = -\delta \cdot g \cdot y + p_0} \quad (6^{**})$$

$\cdot)$ Se define: $\Delta p_{AB} = p(A) - p(B)$

$$\Delta p_{AB} = p(1,0) - p(0,1) = [-\delta g \cdot 0 + p_0] - [-\delta g \, 1 + p_0]$$

$$\Delta p_{AB} = 0 - -\delta \cdot g \cdot 1 + p_0 - p_0$$

$$\boxed{\Delta p_{AB} = \delta \cdot g}$$

Problema 7

Las componentes de la velocidad u y v de un flujo bidimensional están dadas por:

$$u = ax + \frac{b}{xy^2},$$

$$v = -\left(ay + \frac{b}{x^2 y}\right)$$

Siendo a y b constantes.

a. Hallar la aceleración lineal en el punto P = (1,0)
b. Hallar la función corriente. Indicar cuáles son las características salientes de la función corriente.
c. Explicar, para un flujo ideal, qué relación hay entre las curvas de nivel de ψ y las curvas de nivel de ϕ. justificar.

a) Hallar aceleración lineal en (1, 0)

$$a = \frac{D\bar{V}}{DT} = \frac{\partial \bar{V}}{\partial T} + u \cdot \frac{\partial \bar{V}}{\partial x} + V \cdot \frac{\partial \bar{V}}{\partial y}$$

$$\bar{a} = \bar{0} + \left(ax + \frac{b}{xy^2}\right) \cdot \left(a + \frac{b}{y^2} \cdot (-1) \cdot x^{-2} ; \frac{-b}{y} \cdot -2x^{-3}\right)$$

$$+ \left(-ay - \frac{b}{x^2 y}\right) \cdot \left(\frac{b}{x} \cdot -2y^{-3} ; -a - \frac{b}{x^2} \cdot -1 \cdot y^{-2}\right)$$

$$\bar{a} = \left[\left(ax + \frac{b}{xy^2}\right) \cdot \left(a - \frac{b}{x^2 y^2} ; \frac{2b}{y \cdot x^3}\right)\right] + \left[\left(-ay - \frac{b}{x^2 y}\right)\left(\frac{-2b}{xy^3} ; -a + \frac{b}{x^2 y^2}\right)\right]$$

$$\bar{a} = \left(a^2 x - \cancel{\frac{abx}{x^2 y^2}} + \cancel{\frac{ab}{xy^2}} - \frac{b^2}{x^3 y^4} ; \frac{2abx}{y \cdot x^3} + \frac{2b^2}{x^4 y^3}\right) +$$

$$+ \left(\frac{2aby}{x \cdot y^3} + \frac{2b^2}{x^3 y^4} ; a^2 y - \frac{b^2}{x^4 y^3} - \cancel{\frac{bay}{x^2 y^2}} + \cancel{\frac{ab}{x^2 y}}\right)$$

$$\bar{a} = \left(a^2 x - \frac{b^2}{x^3 y^4} ; \frac{2ab}{yx^2} + \frac{2b^2}{x^4 y^3}\right) + \left(\frac{2ab}{x y^2} + \frac{2b^2}{x^3 y^4} , a^2 y - \frac{b^3}{x^4 y^3}\right)$$

$$\boxed{\bar{a} = \left(a^2 x + \frac{2ab}{xy^2} + \frac{b^2}{x^3 y^3} ; \frac{2ab}{x^2 y} + a^2 y + \frac{b^2}{x^4 y^3}\right)} \; (1)$$

Según (1), Dominio $(\bar{v})$: $\mathbb{R}^2 - \{x = 0 ; y = 0\}$

no aplica calcular $\bar{V}$ ni $\bar{a}$ en $(1, 0)$

b) Función corriente ψ

$\boxed{u = \frac{\partial \psi}{\partial y}}$ (3) $\boxed{v = -\frac{\partial \psi}{\partial x}}$ (4)

de (3); $\quad \partial\psi = u \cdot dy \rightarrow \psi = \int \left(ax + \frac{b}{xy^2}\right) dy$

$$\psi = axy + \frac{b}{x}\int y^{-2}\,dy + C(x),$$

$$\psi = axy + \frac{b}{x}\cdot -1 \cdot y^{-1} + C(x) \rightarrow \boxed{\psi = axy - b/yx + C(x)} \quad (5)$$

de (4); $\quad \frac{\partial \psi}{\partial x} = -v \longrightarrow \begin{cases} \frac{\partial \psi}{\partial x} = ay - \frac{b}{y}\cdot -1 \cdot x^{-2} + C'(x) \\ \frac{\partial \psi}{\partial x} = -v = ay + \frac{b}{x^2 y} \end{cases}$

$$\cancel{ay} + \cancel{\frac{b}{x^2 y}} + C'(x) = \cancel{ay} + \cancel{\frac{b}{x^2 y}}, \quad C'(x) = 0, \quad \boxed{C(x) = K}$$

$K \in \mathbb{R}$,

queda finalmente, $\boxed{\psi = axy - \frac{b}{xy} + K}$

c) para flujo ideal; según ψ y ϕ:

líneas de ψ y ϕ son mutuamente ortogonales; según White ec 4.128 (pág 244):

$$\boxed{\left(\frac{dy}{dx}\right)_{\phi = const} = -\frac{u}{v} = -\frac{1}{(dy/dx)_{\psi = const}}}$$

Problema 8

a. Hallar la ecuación de la línea de corriente (línea de campo) que pasa por el punto (x_0, y_0,0) y el vector aceleración del siguiente campo de velocidades:

$$V = (x(1+3t), y, 0)$$

b. ¿Se trata de un fluido ideal? Justificar

Problema 8 a) Hallar línea de corriente con:

$\bar{V}: \mathbb{R}^4 \longrightarrow \mathbb{R}^3 : \boxed{\bar{V}(x, y, z, t) = (x(1+3t); y; 0)}$ ①

en $\boxed{(x_0; y_0; 0)}$ ②

·) Se sabe: $\bar{V} = \frac{d\bar{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = (u, v, w)$ ∴

·) $\frac{dx}{dt} = u \longrightarrow \boxed{dt = \frac{dx}{u}}$ ③; $\frac{dy}{dt} = v \longrightarrow \boxed{dt = \frac{dy}{v}}$ ④

·) de ③ y ④: $\frac{dx}{u} = \frac{dy}{v} \Rightarrow \int \frac{dx}{u} = \int \frac{dy}{v} \Rightarrow$

$\Rightarrow \int \frac{dx}{x(1+3t)} = \int \frac{dy}{y} \Rightarrow \frac{1}{(1+3t)} \int \frac{dx}{x} = \int \frac{dy}{y} \Rightarrow$

$\Rightarrow \frac{1}{(1+3t)} \cdot \ln|x| = \ln|y| + \ln|C| \Rightarrow$

$\Rightarrow \frac{1}{(1+3t)} \cdot \ln|x| = \ln|y \cdot C| \Rightarrow$

$\Rightarrow e^{\left[\frac{1}{1+3t} \cdot \ln|x|\right]} = e^{\ln|y \cdot C|} \Rightarrow$

$\Rightarrow |x|^{\left(\frac{1}{1+3t}\right)} = |y \cdot C| \Rightarrow |K| = \frac{1}{|C|}$

$\Rightarrow \boxed{|y| = |K| \cdot |x|^{\left(\frac{1}{1+3t}\right)}}$ ⑤ Busco $|K|$:

$$|y_0| = |K| \cdot |x_0|^{(1/1+3t)} \rightarrow$$

$$\rightarrow \quad |K| = \frac{|y_0|}{|x_0|^{(1/1+3t)}} \Rightarrow |y_0| \cdot \left[|x_0|^{(1/1+3t)}\right]^{-1}$$

$$\boxed{|K| = |y_0| \, |x_0|^{\frac{-1}{(1+3t)}}} \; (6) \quad \therefore \text{ queda:}$$

$$|y| = |y_0| \cdot |x_0|^{(-1/1+3t)} \cdot |x|^{(1/1+3t)}$$

$$\boxed{|y| = |y_0| \cdot \left|\frac{x}{x_0}\right|^{\left(\frac{1}{1+3t}\right)}} \; (5^+)$$

(c) ¿Vector aceleración $\bar{a}$? Utilizo ec. Euler Lagrange con $w=0$

$$\bar{a} = \frac{D\bar{V}}{Dt} = \frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot (\nabla \cdot \bar{V})$$

$$\bar{a} = \begin{cases} a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \cancel{v \cdot \frac{\partial u}{\partial y}}^{\,0} + \cancel{w \cdot \frac{\partial u}{\partial z}}^{\,0} \\ a_y = \cancel{\frac{\partial v}{\partial t}}^{\,0} + \cancel{u \cdot \frac{\partial v}{\partial x}}^{\,0} + v \frac{\partial v}{\partial y} + \cancel{w \frac{\partial v}{\partial z}}^{\,0} \\ a_z = \cancel{\frac{\partial w}{\partial t}}^{\,0} + \cancel{u \cdot \frac{\partial w}{\partial x}}^{\,0} + \cancel{v \cdot \frac{\partial w}{\partial y}}^{\,0} + \cancel{w \frac{\partial w}{\partial z}}^{\,0} \end{cases}$$

queda: $\bar{a} = \left(\frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} \; ; \; v \cdot \frac{\partial v}{\partial y}\right)$, calculo por partes:

$$) \; \frac{\partial u}{\partial t} = \frac{\partial(x(1+3t))}{\partial t} = x \cdot \frac{\partial(1+3t)}{\partial t} = x\left(\frac{\partial 1}{\partial t} + \frac{\partial (3t)}{\partial t}\right)$$

$$\frac{\partial u}{\partial t} = x(0+3) \longrightarrow \boxed{\frac{\partial u}{\partial x} = 3x}$$

·) $$\left[\frac{\partial u}{\partial x} = \frac{\partial(x(1+3t))}{\partial x} = (1+3t)\right]$$ reemplazo en $\bar{a}$

·) $$\bar{a} = \left(3x + x(1+3t)\cdot(1+3t)\right)\hat{i}\,; \quad (6)\ \boxed{\bar{a} = 3x + x(1+3t)^2\,\hat{i}}$$

b) ¿Fluido ideal? · Un fluido es ideal si cumple:

$$\boxed{\bar{\nabla} \times \bar{V} = \bar{0}} \quad (7)$$

$$\bar{\nabla} \times \bar{V} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z};\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x};\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\bar{\nabla} \times \bar{V} = \left(0 - 0;\ 0 - 0;\ \frac{\partial(y)}{\partial x} - \frac{\partial(x(1+3t))}{\partial y}\right) \quad \therefore$$

$$\boxed{\bar{\nabla} \times \bar{V} = \bar{0}} \quad ✓$$

Problema 9

Considerar un flujo bidimensional incompresible cuya función potencial es:

$$\phi = xy + x^2 - y^2$$

a. ¿Es cierto que el laplaciano es cero?, si ese es el caso, ¿qué significa?
b. Hallar la función corriente
c. Hallar la ecuación de la línea de campo que pasa por el punto (2,1).

a) Dado $\phi = xy + x^2 - y^2$, laplaciano $(\mathbb{L}(\phi))$

$$\mathbb{L}(\phi) = \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$= \frac{\partial (y + 2x)}{\partial x} + \frac{\partial (x - 2y)}{\partial y} = 0$$

$$= 2 + -2 = 0 \quad \checkmark \Rightarrow \boxed{\mathbb{L}(\phi) = 0}$$

Para laplaciano cero; flujo incompresible, fluido ideal

Existe funcion corriente.

b) $$\begin{cases} u = \dfrac{d\psi}{dy}, \quad v = -\dfrac{\partial \psi}{\partial x} & ;① \\ u = \dfrac{d\phi}{dx}, \quad v = \dfrac{d\phi}{dy} & ② \end{cases}$$

de ①: $$\frac{d\phi}{dx} = y + 2x = \frac{d\psi}{dy}$$

$$\int_0^{\psi} d\psi = \int_0^{y} (y + 2x) \cdot dy$$

$$\boxed{\psi = \frac{y^2}{2} + 2xy + f(x)} \quad ③$$

de ②: $$\frac{d\phi}{dy} = x - 2y = \frac{d\psi}{dx}; \quad -\frac{d\psi}{dx} = -2y - f'(x)$$

$$x - \cancel{2y} = -\cancel{2y} - f'(x)$$

$$f'(x) = -x \longrightarrow \boxed{f(x) = -\frac{x^2}{2} + C} \quad (4)$$

Finalmente: $$\boxed{\psi = \frac{y^2}{2} + 2xy - \frac{x^2}{2} + C} \quad (5)$$

verifico ψ

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial\left[\frac{y^2}{2} + 2xy - \frac{x^2}{2} + C\right]}{\partial y} = y + 2x$$

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial\left(xy + x^2 - y^2\right)}{\partial x} = y + 2x$$

✓

$$V = -\frac{\partial \psi}{\partial x} = (-1)\cdot \frac{d\left(\frac{y^2}{2} + 2xy - \frac{x^2}{2} + C\right)}{\partial x} = -2y + x$$

$$V = \frac{\partial \phi}{\partial y} = \frac{\partial\left(xy + x^2 - y^2\right)}{\partial y} = x - 2y$$

✓

Problema 10

a) Decidir si es posible el siguiente campo de flujo:

$$u = V\,(x^3 + xy^2)$$
$$v = V\,(y^3 + yx^2)$$

Donde V es una constante.

b) Un campo de flujo bidimensional está definido por:

$$u = xt + 2y\ ;\ \ v = xt^2 - yt$$

¿Cuál es la aceleración en el punto (1, 1) m , para t = 1s? ¿El flujo es rotacional o irrotacional?

a) $\boxed{\bar{V} = V\,(x^3 + xy^2;\ y^3 + yx^2)}$

Es incompresible? $\Leftrightarrow$ $\frac{du}{dx} + \frac{dv}{\partial y} = 0$

$V\left[(3x^2 + y^2) + (3y^2 + x^2)\right] = 0 \to$ no

¿Es irrotacional? $\Leftrightarrow$ $\bar{\nabla} \times \bar{v} = 0$

$$\bar{\nabla} \times \bar{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V(x^3 + xy^2) & V(y^3 + yx^2) & 0 \end{vmatrix} = (0, 0, 2xy - 2xy)$$

$\boxed{\bar{\nabla} \times \bar{v} = \bar{0}} \to$ por esta condición, $\exists\ \phi$ tal que $\boxed{\bar{V} = \bar{\nabla}\phi}$

b) $\bar{V} = (x\pi + zy;\ x\pi^2 - y\pi)$

Calculo $\bar{\nabla} \times \bar{v} \to$ $\bar{\nabla} \times \bar{v} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x\pi + zy & x\pi^2 - y\pi & 0 \end{vmatrix} =$

$\boxed{\bar{\nabla} \times \bar{v} = (0, 0, \pi^2 - z)}$

Se cumple $(\bar{\nabla} \times \bar{v} = \bar{0})$ (Flujo irrotacional) $\Leftrightarrow$ $\pi = \sqrt{z}$

<u>Justificación</u>: el campo es irrotacional sólo en el instante $\pi = \sqrt{z}$, para todo $\pi \neq \sqrt{z}$ es rotacional

En algún ejemplo en el que se cumpla $\bar{\nabla} \times \bar{v} = \bar{0}$ para <u>algunos puntos</u> (x_1, y_1), es <u>falso</u> el concepto de campo irrotacional en dichos puntos. El campo es rotacional o irrotacional como un todo, no tiene sentido la aplicación para puntos aislados.

$$\bar{a} = \frac{\partial \bar{v}}{\partial t} + u \frac{\partial \bar{v}}{\partial x} + v \frac{\partial \bar{v}}{\partial y}$$

$$\bar{a} = (x,\ 2xt - y) + (xt + 2y)(t,\ t^2) + (xt^2 - yt)(2,\ -t)$$

$$\bar{a} = (x,\ 2xt - y) + (xt^2 + 2yt,\ xt^3 + 2yt^2) + (2xt^2 - yt,\ -xt^3 + yt^2)$$

para $\bar{a}(1,1)$ $t = t$

$$\bar{a} = (1,\ 2t - 1) + (t^2 + 2t,\ t^3 + 2t^2) + (2t^2 - 2t,\ -t^3 + t^2)$$

$$\bar{a} = (1 + t^2 + \cancel{2t} + 2t^2 - \cancel{2t},\ 2t - 1 + \cancel{t^3} + 2t^2 - \cancel{t^3} + t^2)$$

$$\boxed{\bar{a} = (3t^2 + 1,\ 3t^2 + 2t - 1)}$$

para $\bar{u}(1,1)$; $t = 1$

$$\bar{a} = (3 \cdot 1^2 + 1,\ 3 \cdot 1^2 + 2 \cdot 1 - 1)$$

$$\bar{a} = (3 + 1,\ 3 + 2 - 1); \quad \boxed{\bar{a} = (4,\ 4)}$$

Problema 11

Una componente u de un flujo incompresible de velocidad está dada por u = Axy, donde A es una constate ¿Cuál es una posible componente v? ¿Cuál debe ser la componente v si el flujo es irrotacional?

Fluso $\xrightarrow{a)}$ incompresible $\longrightarrow$ $\frac{du}{dx} + \frac{dv}{dy} = 0$ (1)

↳ b) irrotacional $\longrightarrow$ $\bar{\nabla} \times \bar{v} = \bar{0}$ (2)

a (1)
$$\frac{d(Axy)}{dx} + \frac{dv}{dy} = 0$$

$$Ay + \frac{dv}{dy} = 0 \longrightarrow \frac{dv}{dy} = -Ay$$

$$dv = -A \cdot y \cdot dy$$

$$\boxed{V = -\frac{A}{2} y^2 + f(x)} \quad (1)$$

a (2):
$$\bar{\nabla} \times \bar{v} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & 0 \end{vmatrix} =$$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Axy & V & 0 \end{vmatrix} = (0, 0, 0)$$

$$\frac{\partial(0)}{\partial y} - \frac{\partial V}{\partial z} = 0 \longrightarrow \frac{\partial V}{\partial z} = 0$$

↳ $V = f(x, y)$

Tomando $v = \frac{-A}{2} y^2 + f(x)$

$$\frac{\partial v}{\partial x} - \frac{\partial (Axy)}{\partial y} = 0$$

$$f'(x) - Ax = 0 \longrightarrow f'(x) = Ax$$

$$\frac{df}{dx} = Ax$$

$$\int_0^f df = \int_0^x Ax\,dx$$

$$\boxed{f = \frac{A}{2} x^2 + C}$$, queda:

$$\boxed{\bar{V} = \left(Axy ; \frac{-A}{2} y^2 + \frac{A}{2} x^2 + C \right)}$$ (3)

Validación:) es incompresible $\Leftrightarrow$ $\bar{\nabla} \cdot \bar{V} = 0$.

$$\bar{\nabla} \cdot \bar{V} = \left(\frac{\partial}{\partial x} , \frac{\partial}{\partial y} \right) \cdot (u, v) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \Rightarrow$$

$$\Rightarrow \frac{\partial (Axy)}{\partial x} + \frac{\partial \left(-\frac{A}{2} y^2 + \frac{A}{2} x^2 + C \right)}{\partial y} = Ay - Ay = 0 \text{ OK}$$

) es irrotacional $\Leftrightarrow$ $\bar{\nabla} \times \bar{V} = \bar{0}$

$$\bar{\nabla} \times \bar{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\cancel{\frac{\partial w}{\partial y}}^{0} - \cancel{\frac{\partial v}{\partial z}}^{0} ; \cancel{\frac{\partial u}{\partial z}}^{0} - \cancel{\frac{\partial w}{\partial x}}^{0} ; \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\bar{\nabla} \times \bar{V} = \left(0; 0; \frac{\partial \left(-\frac{A}{2} y^2 + \frac{A}{2} x^2 + C \right)}{\partial x} - \frac{\partial (Axy)}{\partial y} \right) \Rightarrow$$

$$\Rightarrow \bar{\nabla} \times \bar{V} = (0; 0; Ax - Ax) \Rightarrow \boxed{\bar{\nabla} \times \bar{V} = \bar{0}}$$ ✓

Problema 12

Un fluido tiene un campo de temperaturas dado por la siguiente expresión:

$T = 50°C - 2\ {}^{°C}/_{m^2}\,(x^2 + y^2)$

La misma se encuentra en un espacio cuyo campo de velocidades es $V(x, y, z) = e^{-t}(2x, -2y, 0)\ {}^{m}/_{s}$

Siendo el punto *P (1,1,0)*, se pide determinar:

a. ¿Cuál es la rapidez de cambio de la temperatura para una partícula que en t = 0 está en P?

b. ¿Qué aceleración tiene esa particula?

c. ¿Cuánto vale la vorticidad para la misma partícula?

d. ¿Cuánto vale la vorticidad en un punto cualquiera de este campo?

e. ¿Cuánto vale la divergencia del campo de velocidades?

Problema 12

a) Hallar $DT/D\tau$ para $\tau = 0$ y $P = (1,1,0)$

T: temperatura ; τ: tiempo

De derivada total:

$$\boxed{\frac{DT}{D\tau} = \frac{\partial T}{\partial \tau} + \bar{V}(\bar{\nabla} \cdot T)} \quad (1) ; \qquad T: \mathbb{R}^4 \longrightarrow \mathbb{R}$$

$$\boxed{T(x,y,z,\tau) = 50°C - \frac{2°C}{m^2}(x^2+y^2)} \quad (2)$$

$$\bar{V}: \mathbb{R}^4 \longrightarrow \mathbb{R}, \qquad \boxed{\bar{V}(x,y,z,\tau) = e^{-\tau}(2x; -2y; 0)\ m/s} \quad (3)$$

(+) de (3), (2) y (1):

$$\cdot) \left[\frac{\partial T}{\partial \tau} = \frac{\partial \left(50 - 2(x^2+y^2)\right)}{\partial \tau} = 0 \right]$$

$$\cdot) \left[\bar{\nabla} \cdot T = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right) \cdot T = \left(\frac{\partial T}{\partial x}; \frac{\partial T}{\partial y}; \frac{\partial T}{\partial z}\right) \right]$$

$$\cdot) \left[\bar{V} \cdot (\bar{\nabla} \cdot T) = (u, v, w) \cdot \left(\frac{\partial T}{\partial x}; \frac{\partial T}{\partial y}; \frac{\partial T}{\partial z}\right) = \right.$$

$$\bar{V}(\bar{\nabla} T) = u \cdot \frac{\partial T}{\partial x} + v \cdot \frac{\partial T}{\partial y} + w \cdot \cancel{\frac{\partial T}{\partial z}}^{0} \quad \therefore \text{queda:}$$

$$\boxed{\frac{DT}{D\tau} = 0 + u \cdot \frac{dT}{dx} + v \cdot \frac{dT}{dy}}$$

$$\cdot)\ u \cdot \frac{dT}{dx} = e^{-\tau} \cdot 2x \cdot \frac{d\left(50 - 2(x^2 + y^2)\right)}{\partial x} = 2xe^{-\tau}\left[\frac{d(50)}{dx} - \frac{2d(x^2+y^2)}{\partial x}\right]$$

$$u\frac{dT}{dx} = 2xe^{-\tau}\left(0 - 2 \cdot 2 \cdot x + 0\right) ; \quad \boxed{u\frac{dT}{dx} = -8x^2e^{-\tau}}$$

$$\cdot)\ v \cdot \frac{dT}{dy} = -e^{-\tau} 2y \cdot \frac{d\left(50 - 2x^2 - y^2\right)}{\partial y} \Rightarrow \boxed{v \cdot \frac{dT}{dy} = 8y^2e^{-\tau}}$$

queda: $\boxed{\frac{DT}{D\tau} = 8e^{-\tau}\left(y^2 - x^2\right)}$ (4) (8)

en el punto P y τ: $\frac{DT}{D\tau} = 8 \cdot e^{-0} \cdot (1^2 - 1^2) = 8 \cdot 1 \cdot 0$; $\boxed{\frac{DT}{D\tau} = 0}$

b) Hallar aceleración $\bar{a}$; del concepto de derivada total

$$\bar{a} = \frac{D\bar{V}}{D\tau} = \frac{\partial \bar{V}}{\partial \tau} + \bar{V} \cdot (\bar{\nabla} \cdot \bar{V}), \quad \text{con } w = 0$$

$$\bar{a} \begin{cases} a_x = \dfrac{\partial u}{\partial \tau} + u\dfrac{\partial u}{\partial x} + v \cdot \cancel{\dfrac{\partial u}{\partial y}}^{\,0} + \cancel{w \cdot \dfrac{\partial u}{\partial z}}^{\,0} \\ a_y = \dfrac{\partial v}{\partial \tau} + \cancel{u \cdot \dfrac{\partial v}{\partial x}}^{\,0} + v \cdot \dfrac{\partial v}{\partial y} + \cancel{w \cdot \dfrac{\partial v}{\partial z}}^{\,0} \\ a_z = \cancel{\dfrac{\partial w}{\partial \tau}}^{\,0} + \cancel{u\dfrac{\partial w}{\partial x}}^{\,0} + \cancel{v \cdot \dfrac{\partial w}{\partial y}}^{\,0} + \cancel{w \cdot \dfrac{\partial w}{\partial z}}^{\,0} \end{cases}$$

Calculemos las componentes:

$$\cdot)\ \frac{\partial u}{\partial t} = \frac{\partial(2xe^{-t})}{\partial t} = 2xe^{-t} \qquad \cdot)\ \frac{\partial v}{\partial t} = \frac{\partial(-2ye^{-t})}{\partial t} = -2ye^{-t}$$

$$\cdot)\ \frac{\partial u}{\partial x} = \frac{\partial(e^{-t}\cdot 2x)}{\partial x} = 2e^{-t} \qquad \cdot)\ \frac{\partial u}{\partial y} = \frac{\partial(e^{-t}\cdot 2x)}{\partial y} = 0$$

$$\cdot)\ \frac{\partial v}{\partial x} = \frac{\partial(-2y\cdot e^{-t})}{\partial x} = 0 \qquad \cdot)\ \frac{\partial v}{\partial y} = \frac{\partial(-2ye^{-t})}{\partial y} = -2e^{-t}$$

$$\bar{a} = \left(\frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x}\right)\hat{i} + \left(\frac{\partial v}{\partial t} + v\cdot\frac{\partial v}{\partial y}\right)\hat{j}$$

$$\bar{a} = \left(2xe^{-t} + 2x\cdot e^{-t}\cdot 2\cdot e^{-t}\right)\hat{i} + \left(-2ye^{-t} - 2y\, e^{-t}(-2e^{-t})\right)\hat{j}$$

$$\boxed{\bar{a} = \left(2xe^{-t} + 4xe^{-2t}\right)\hat{i} - \left(2ye^{-t} - 4y\,e^{-2t}\right)\hat{j}} \quad \begin{array}{l}\to P=(1,1,0)\\ \to t=0\end{array}$$

$$\bar{a} = (2\cdot1\cdot1 + 4\cdot1\cdot1)\hat{i} - (2\cdot1\cdot1 - 4\cdot1\cdot1)\hat{j} \;\therefore\; \boxed{\bar{a} = 6\hat{i} + 2\hat{j}} \quad (5)$$

c) ¿Vorticidad? Se define a la vorticidad $\bar{\zeta}$ como

$$\boxed{\bar{\zeta} = \bar{\nabla}\times\bar{V}} \quad (6)$$

Aclaración: $\bar{\zeta}$: notación de White

$$\bar{\zeta} = \bar{\nabla}\times\bar{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\cancel{\frac{\partial w}{\partial y}}^{\,0} - \cancel{\frac{\partial v}{\partial z}}^{\,0};\ \cancel{\frac{\partial u}{\partial z}}^{\,0} - \cancel{\frac{\partial w}{\partial x}}^{\,0};\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\bar{\zeta} = \left(0, 0;\ \cancel{\frac{\partial(-2ye^{-t})}{\partial x}}^{\,0} - \cancel{\frac{\partial(2xe^{-t})}{\partial y}}^{\,0}\right), \quad \boxed{\bar{\zeta} = \bar{0}} \quad (7)$$

e) ¿Divergencia? Se define: $\boxed{\operatorname{div}(\bar{V}) = \bar{\nabla}\cdot\bar{V}}$ (8)

$$div(\bar{v}) = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right)\cdot(u, v, w) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}}^{0}$$

$$div(\bar{v}) = \frac{\partial(2xe^{-t})}{\partial x} + \frac{\partial(-2ye^{-t})}{\partial y} = 2e^{-t} - 2e^{-t}$$

$$\therefore \boxed{div(\bar{v}) = 0}$$

Problema 14

Un campo bidimensional de velocidades de flujo tiene las componentes *u* = *A(x2-y2)*; *v* = *-2Axy*. Determinar:

a. Si el flujo es compresible o incompresible (justificar);
b. Si el flujo es estacionario o no estacionario;
c. La aceleración de una partícula en un punto cualquiera.

$$\boxed{\bar{V} = (A(x^2 - y^2), -2Axy)} \quad (1)$$

a) Flujo incompresible $\Leftrightarrow$ $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$ (2)

$$\frac{\partial(A(x^2-y^2))}{\partial x} + \frac{\partial(-2Axy)}{\partial y} = 0$$

$$2AX - 2AX = 0 \longrightarrow \text{incompresible}$$

b) Flujo estacionario $\Leftrightarrow$ $\frac{d\bar{v}}{dt} = \bar{0}$

$$\boxed{\frac{d\bar{v}}{dt} = (0, 0)}$$

c) aceleración

$$\bar{a} = \frac{d\bar{V}}{dt} + u\,\frac{\partial \bar{V}}{\partial x} + v\,\frac{\partial \bar{V}}{\partial y}$$

$$\bar{a} = (0,0) + \left[A(x^2-y^2)\cdot(2Ax, -2Ay)\right] +$$

$$+ \left[-2Axy\cdot(-2Ay\,; -2Ax)\right]$$

$$\bar{a} = \left(2A^2x^2(x^2-y^2)\,; -2A^2y(x^2-y^2)\right) +$$

$$+ \left(4A^2xy^2, 4A^2x^2y\right)$$

$$\boxed{\bar{a} = \left(2A^2x^4 - 2A^2x^2y^2 + 4A^2xy^2\,; -2A^2x^2y + 2A^2y^3 + 4A^2x^2y\right)}$$

Problema 15

Ídem ejercicio 14, pero si ahora el campo de velocidades está dado por

$$u = x^2 - y^2,\ v = -xy$$

$$\boxed{\bar{V} = (x^2 - y^2;\ -xy)} \quad 1$$

a) $\dfrac{du}{dx} + \dfrac{dv}{dy} = 0$?

$2x - x \neq 0 \rightarrow$ compresible

b) $\dfrac{d\bar{V}}{dt} = (0,0) \rightarrow$ estacionario

c) $\bar{a} = \bar{0} + (x^2 - y^2) \cdot (2x, -y) + (-xy) \cdot (-y, -x)$

$\bar{a} = (2x^3 - 2xy^2,\ -yx^2 + y^3) + (2xy^2,\ x^2 y)$

$$\boxed{\bar{a} = (2x^3,\ y^3)}$$