

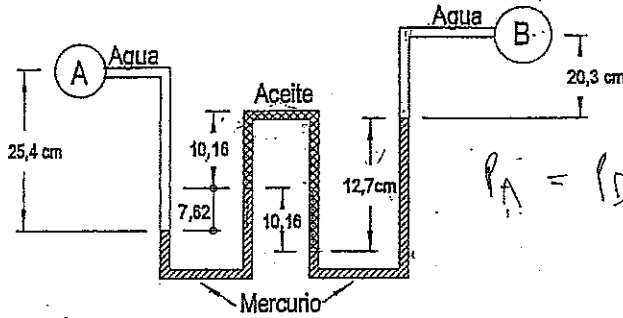
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PARTE PRÁCTICA

Problema 1:

Para el manómetro de la figura con aceite de densidad relativa 0,8, mercurio de densidad relativa 13,6 y agua de densidad relativa 1, calcular la diferencia de presiones entre los puntos A y B. Las medidas están en centímetros.

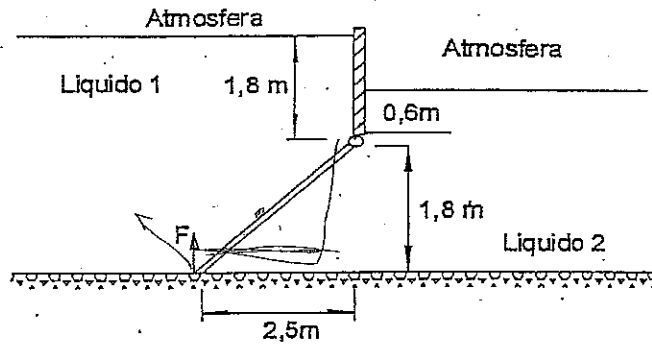


$$P_A = P_B + 12.7 \gamma - 10.16 \gamma_{ac} +$$

Problema 2:

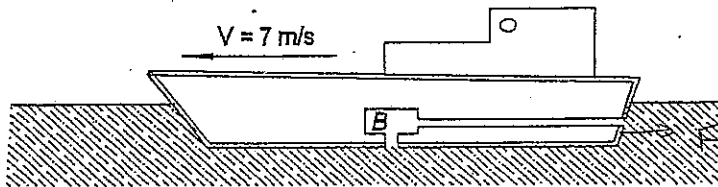
Encontrar la magnitud de las fuerzas de cada lado de la compuerta rectangular de 1,5 m de ancho. Encontrar el centro de presión de las fuerzas de cada lado de la misma. Determinar la fuerza F necesaria para que la compuerta pueda abrirse, la compuerta 1500 Kg. M. de inercia del rectángulo = $b \cdot h^3 / 12$

Datos: Líquido₁ $\gamma_1 = 1000 \text{ kg/m}^3$ -
Líquido₂ $\gamma_2 = 800 \text{ kg/m}^3$



Problema 3: (entra?)

Un barco es propulsado a 7 m/s mediante una bomba que toma agua del centro del barco. La descarga en la popa se hace mediante un conducto de 15 cm de diámetro con un caudal de $0.4 \text{ m}^3/\text{s}$. ¿Qué fuerza de propulsión se ejercerá sobre el barco sobre el agua? ¿Cuál será la potencia puesta en juego para esta propulsión?



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$$\psi(x,y) = \frac{y^2}{2} - \frac{x^2}{2} + 2xy$$

Otra forma

$$\left\{ \begin{array}{l} \frac{\partial \psi}{\partial y} = y + 2x \longrightarrow \psi(x,y) = \frac{y^2}{2} + 2xy + f(x) \\ \frac{\partial \psi}{\partial x} = -x + 2y \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial \psi}{\partial x} = -x + 2y + f'(x) = -x + 2y \\ f'(x) = -x \\ f(x) = -\frac{x^2}{2} + C \end{array} \right.$$

b) $\vec{v} = (u, v) = (y + 2x, x - 2y)$

$$v(1,2) = (2+2, 1-4) = (4, -3)$$

$$\|\vec{v}\| = \sqrt{16+9} = 5 \quad P(1,2)$$

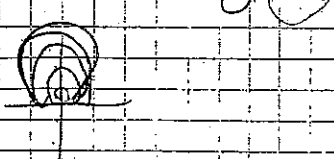
c) Permanente

Compresibilidad ($\text{div}(\vec{v}) = 0 \rightarrow$ incompresible)

$$\text{div}(\vec{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 2 - 2 = 0 \rightarrow \text{incompresible}$$

Rotor

$$\text{rot}(\vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & 0 \end{vmatrix} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 1 - 1 = 0 \rightarrow \text{irrotacional} \therefore \text{lamina}$$



Períod Mecânica de Fluidos (30/8/09)

Parte Teórica:

Problema 1:

$$u = \frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$$
$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Cond. Cauchy-Schwartz

(ϕ potencial, ψ corrente)

$$\phi = xy + x^2 - y^2$$

Potencial \rightarrow Componentes (campo $\vec{v} = (u, v)$) \rightarrow Corrente

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial (xy + x^2 - y^2)}{\partial x} = \boxed{y + 2x}$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial (xy + x^2 - y^2)}{\partial y} = \boxed{x - 2y}$$

$$u = \frac{\partial \psi}{\partial y}$$

$$\partial \psi = u \cdot dy = \int (y + 2x) dy = \frac{y^2}{2} + 2xy + f_1(x)$$

$$\psi_x = \frac{y^2}{2} + 2xy + f(x)$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$\partial \psi = -\partial x \cdot v = -\int (x - 2y) dx = -\frac{x^2}{2} + 2yx + f_2(y)$$

$$\psi_y = \frac{-x^2}{2} + 2yx + f(y) \implies f(y) = \frac{y^2}{2} - y + f(x) = -\frac{x^2}{2}$$

Probleme 2:

a) $\vec{v} = 4xy^2 \vec{i} + f(y) \vec{j} - z^2 \vec{k}$

$\text{div}(\vec{v}) = 0, \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ Ec. dif. continue

$\text{div}(\vec{v}) = \frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 4y^2 + f'(y) - y^2$

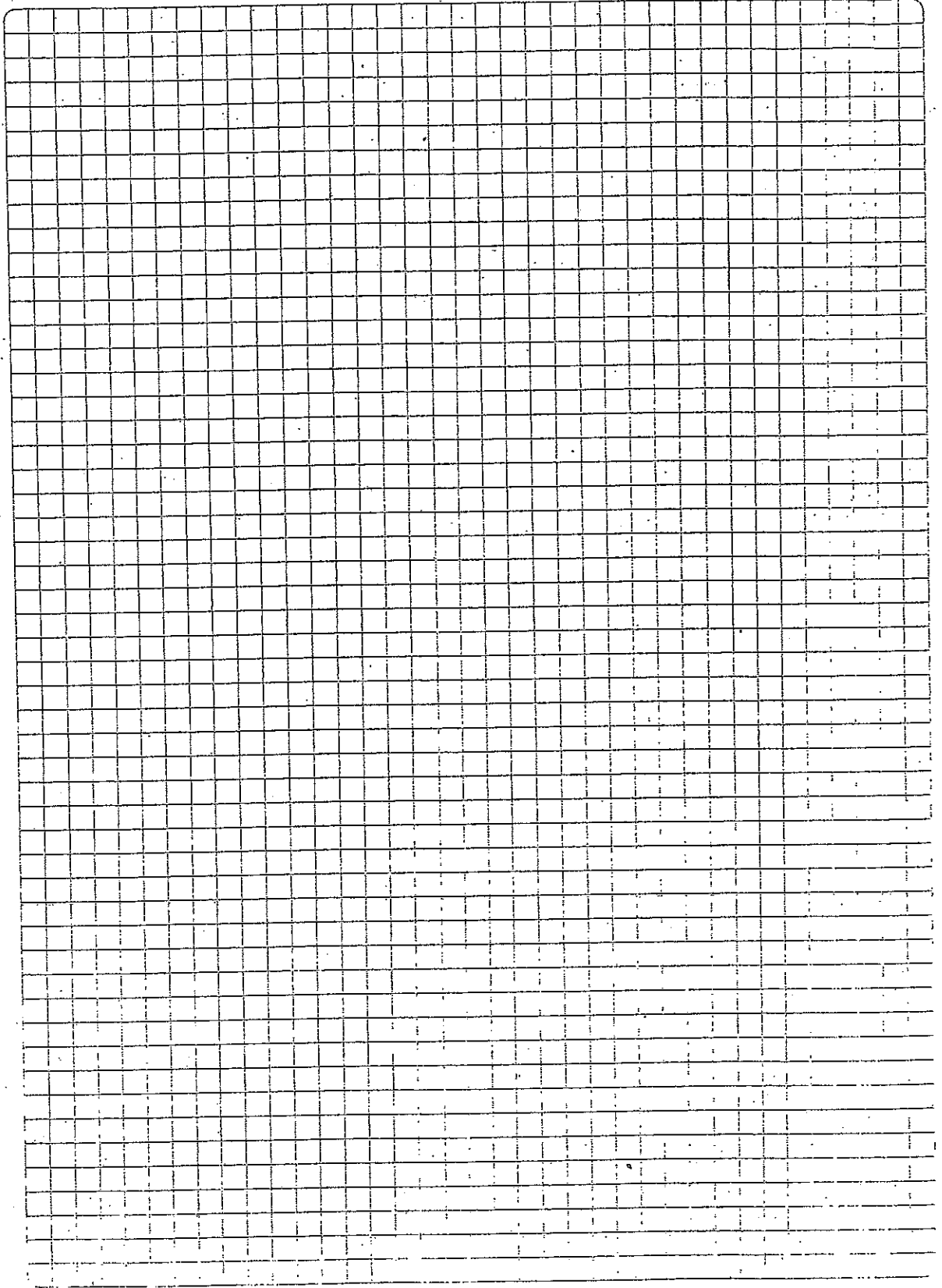
$\text{div}(\vec{v}) = 0$ (Par que campo irrotacional)

$4y^2 + f'(y) - y^2 = 0$

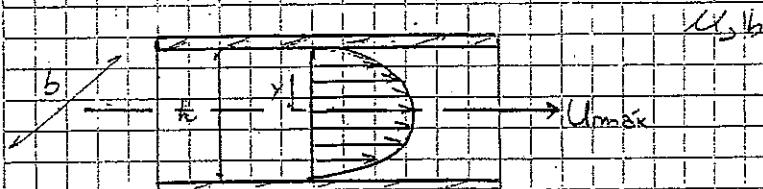
$f'(y) = y^2 - 4y^2 = -3y^2$

$f(y) = -y^3 + f(x,z)$

b)



Problema 3

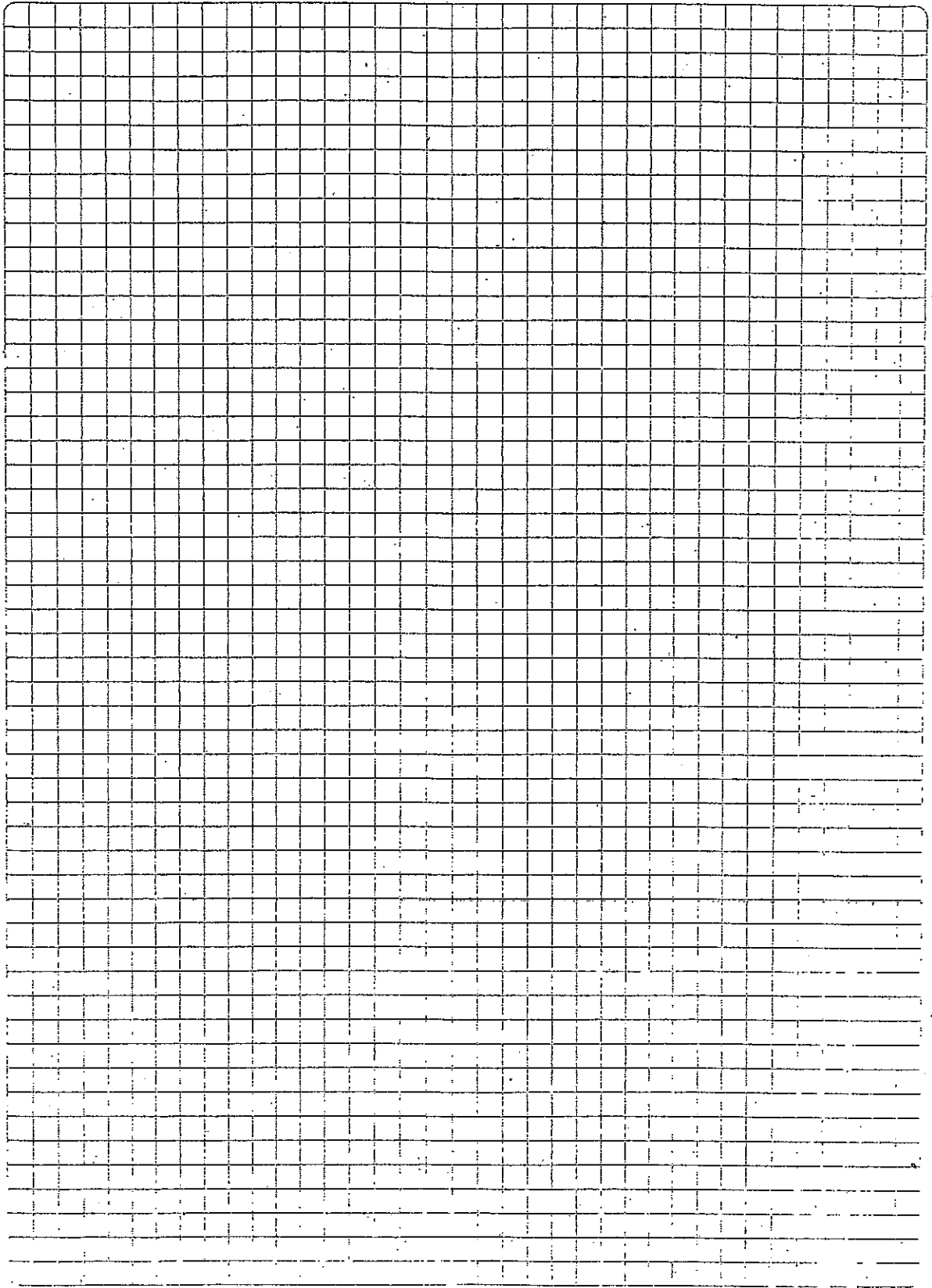


$$u(y) = U_{\max} \left[\frac{y}{r} - \left(\frac{y}{r} \right)^2 \right]$$

$$F = \tau A = \mu \frac{du(y)}{dy} A = \mu \left\{ 2U_{\max} \left(\frac{1}{r} - 2 \left(\frac{y}{r} \right) \left(\frac{1}{r} \right) \right) \right\} \cdot \frac{\pi (h)^2}{4}$$

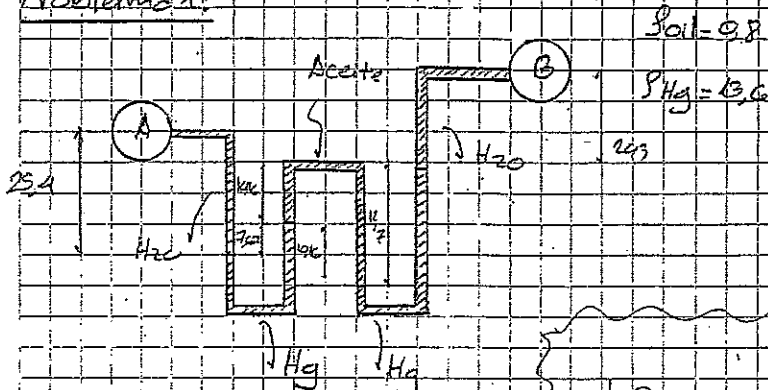
$$F = \mu \cdot 2U_{\max} \left(\frac{1}{r} - \frac{2y}{r^2} \right) \cdot \frac{\pi (h)^2}{4}$$

$$F = \mu \cdot \pi \cdot U_{\max} \cdot h \left(\frac{1 - 2y}{r} \right)$$



Fente Prática

Problema 1:



Oil = 800
 $\rho_{Hg} = 13600$

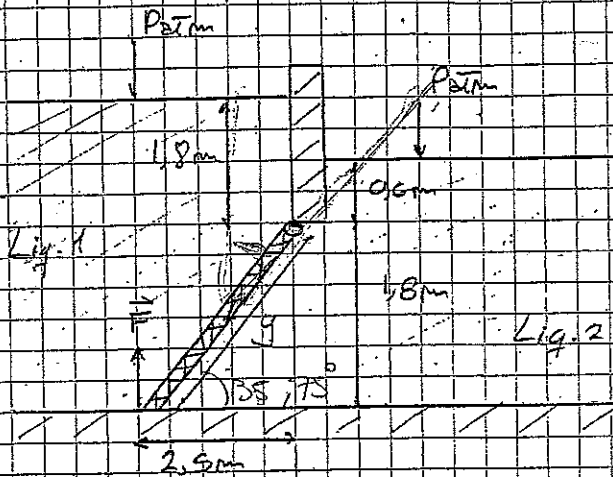
↑ ⊖
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$$P_A - P_B = -\rho_{H_2O} g (0,254 \text{ m}) + \rho_{Ac} g (0,0762 \text{ m}) - \rho_{oil} g (0,106 \text{ m}) + \rho_{Hg} g (0,127 \text{ m}) + \rho_{H_2O} g (0,203 \text{ m})$$

$$\Delta P_{AB} = P_A - P_B = -(1000 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(0,254 \text{ m}) + (13600 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(0,0762 \text{ m}) - (800 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(0,106 \text{ m}) + (13600 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(0,127 \text{ m}) + (1000 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(0,203 \text{ m})$$

$\Delta P_{AB} = P_A - P_B = 25,8 \text{ kPa}$

Problema 2:



$$l = 1,5 \text{ m}$$

$$m = 1500 \text{ kg}$$

$$I_c = \frac{bh^3}{12}$$

$$\rho_1 = 1000 \text{ kg/m}^3 \text{ (P}_1\text{)}$$

$$\rho_2 = 800 \text{ kg/m}^3 \text{ (P}_2\text{)}$$

Liq. 1: (me tengo en cuenta P_{atm} ya que actúa en ambas lados de la pieza, ∴ se cancelan entre sí)

$$F_{B1} = \rho_1 \cdot g \cdot h_c \cdot A = (1000 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(1,8 \text{ m} + 0,9 \text{ m})(l \cdot y)$$

$$y = \sqrt{(1,8 \text{ m})^2 + (2,5 \text{ m})^2} = 3,08 \text{ m}$$

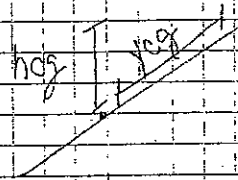
$$F_{B1} = (1000 \text{ kg/m}^3)(9,8 \text{ m/s}^2)(2,7 \text{ m})(1,5 \text{ m} \times 3,08 \text{ m})$$

$$F_{B1} = 122,25 \text{ kN} = 12479 \text{ kgf}$$

$$y_{PI} = \frac{y_c}{y_c \cdot A} + \frac{I_c}{y_c \cdot A} = (2,7 \text{ m}) + \frac{l \cdot y^3}{12(2,7 \text{ m})(1,5 \text{ m} \times 3,08 \text{ m})} =$$

NO e $h_{cg} \neq y_{cg}$!!

$$= (2,7 \text{ m}) + \frac{(1,5 \text{ m})(3,08 \text{ m})^3}{12(2,7 \text{ m})(1,5 \text{ m} \times 3,08 \text{ m})}$$



$$y_{PI} = 3 \text{ m}$$

Antes:

$$\sum \tau_A = 0$$

$$F \cdot x \cdot \sin(\alpha) - F_1 \cos(\alpha) \cdot y_1 + F_2 \cos(\alpha) \cdot y_2 - m \cdot g \cos(\theta) = 0$$

$$F = \frac{m \cdot g \cos(\theta) + F_1 \cos(\alpha) \cdot y_1 - F_2 \cos(\alpha) \cdot y_2}{x \cdot \sin(\alpha)}$$

$$F = \frac{(1500 \text{ kg})(9,8 \text{ m/s}^2) \cdot \cos(35,75) + (122,25 \text{ kN}) \cdot \cos(54,25) \cdot (2,05 \text{ m}) - (54,33 \text{ kN}) \cdot \cos(54,25) \cdot (0,392 \text{ m})}{(3,08 \text{ m}) \cdot \sin(54,25)}$$

$$\boxed{F = 58,35 \text{ kN} = 5959 \text{ kgf}}$$

Fig. 2

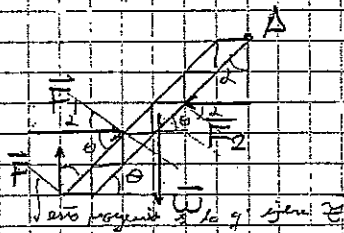
$$F_{R2} = \rho \cdot g \cdot h_c \cdot A = (800 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.6 \text{ m} + 0.9 \text{ m})(1.5 \text{ m} \times 3.08 \text{ m})$$

$$F_{R2} = 54,33 \text{ kN} = 5544 \text{ kgf}$$

$$y_{p2} = y_c + \frac{I_c}{y_c A} = (0.6 \text{ m} + 0.9 \text{ m}) + \frac{(1.5 \text{ m})(3.08 \text{ m})^3}{12(0.6 \text{ m} + 0.9 \text{ m})(1.5 \text{ m})(3.08 \text{ m})}$$

$$y_{p2} = 2,03 \text{ m}$$

b)



Queremos hallar \vec{F}

$$\tan(\theta) = \frac{0.9}{1.5} = \frac{3}{5}$$

$$\theta = 33,75^\circ \rightarrow d = 5.8$$

$\sum \mathcal{E}_A = 0$ (línea de acción de gravedad, cuando se hace el equilibrio)

$$F_1 \cdot \sin(d) - F_2 \cdot \cos(d) \cdot y_1 + F_2 \cdot \cos(d) \cdot y_2 - W \cdot \cos(\theta) = 0$$

$$\cos(d) = \frac{ad}{H} = \frac{(y_{p1} - 1.8 \text{ m})}{y_1}$$

$$y_1 = \frac{(3 \text{ m} - 1.8 \text{ m})}{\cos(33,75^\circ)} = 2.05 \text{ m}$$

$$y_2 \rightarrow \cos(d) = \frac{ad}{H} = \frac{(y_{p2} - 1.8)}{y_2}$$

$$y_2 = \frac{(2.03 \text{ m} - 1.8 \text{ m})}{\cos(33,75^\circ)} = 0.394 \text{ m}$$