Problema 1

(1) $$\boxed{\phi(x,y) = xy + x^2 - y^2} \quad \phi: \mathbb{R}^2 \to \mathbb{R}^2$$ incompresible

a) El Laplaciano de una función $\mathbb{L}(\phi)$ se define:

$$\boxed{\mathbb{L}(\phi) = \nabla^2\phi} \quad (2)$$

Para un fluido con velocidad $\bar{V}: \mathbb{R}^2 \to \mathbb{R}^2$, $\bar{V}(x,y) = (u, v)$ con $u = \frac{d\phi}{dx}$; $v = \frac{d\phi}{dy}$; el flujo es entonces incompresible

Si se cumple:

$$\nabla^2\phi = 0 ; \Rightarrow \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \quad (3)$$

Calculamos (3) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$; $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$

$$\frac{\partial(\phi)}{\partial x} = y + 2x \quad , \quad \frac{\partial\phi}{\partial y} = x - 2y$$

$$\frac{\partial^2\phi}{\partial x^2} = 2 \quad , \quad \frac{\partial\phi}{\partial y} = -2$$, queda:

$$\boxed{2 - 2 = 0} \quad ✓$$

b) Si el flujo es incompresible, existe la función corriente queda:

$$\psi = \int d\psi = \int (y + 2x)\,dy + f(x)$$

$$\boxed{\psi = \frac{y^2}{2} + 2xy + f(x)} \quad (4)$$

Derivo (4); $\cancel{u = -\frac{\partial \psi}{\partial y} = x - 2y}$

$$\cancel{-\frac{\partial \psi}{\partial y} = -y}$$

$$V = -\frac{\partial \psi}{\partial y} = x - 2y$$

$$V = -\frac{\partial \psi}{\partial x} = -2y - f'(x)$$

$$-\cancel{2y} - f'(x) = -\cancel{2y} + x \implies \boxed{\int f'(x)\,dx = -\frac{x^2}{2}}$$

Finalmente queda: $\boxed{\psi = \frac{y^2}{2} + 2xy - \frac{x^2}{2} = C}$ (5)

c) Para el punto $(1,2)$; $\psi(1,2) = \frac{1^2}{2} + 2\cdot 1\cdot 2 - \frac{2^2}{2} = C$

$\cancel{C = 2 + 4 - \frac{1}{2} = 6 - \frac{1}{2} = 5}$

$C = \frac{1}{2} + 4 - 2 = \frac{1}{2} + 2 = \frac{5}{2}$, queda:

$$\frac{y^2}{2} - \frac{x^2}{2} + 2xy = \frac{5}{2}$$

$$\boxed{y^2 + 4xy - x^2 = 5} \quad (6)$$

Es una curva de nivel de función de corriente

Problema 2

$$\bar{V}: \mathbb{R}^3 \to \mathbb{R}^3 \quad , \quad \boxed{\bar{V} = (16x^2+y;\ 10;\ yz^2)}$$

a) hallar $\bar{a}(6,3,2)$

$$\frac{D\bar{V}}{DT} = \bar{a} = \frac{\partial \bar{V}}{\partial T} + \bar{V}(\bar{\nabla}\cdot\bar{V}) \longrightarrow \boxed{\frac{\partial \bar{V}}{\partial T} = \left(\frac{\partial v}{\partial T}, \frac{\partial u}{\partial T}, \frac{\partial w}{\partial T}\right)}$$

$$\cdot)\ a_x = \frac{\partial u}{\partial T} + u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$$

$$a_x = 0 + (16x^2+y)\cdot 32x + 10\cdot 1 + \cancel{y\cdot z^2\ 2zy}\ 0$$

$$a_x = (16\cdot 6^2+3)\cdot 32 + 10^{+0} \cancel{+ 3\cdot 2^2\cdot 2\cdot 2\cdot 3} \quad \cancel{\boxed{a_x = 18682}}$$

$$\boxed{a_x = 18538}$$

$$\cdot)\ a_y = \frac{\partial v}{\partial T} + u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$$

$$a_y = 0 + 0 + 0 + 0, \quad \boxed{a_y = 0}$$

$$\cdot)\ a_z = \frac{\partial w}{\partial T} + u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$$

$$a_z = 0 + 0 + 10\cdot z^2 + y\cdot z^2\cdot 2z\cdot y$$

$$a_z = 10\cdot 2^2 + 3\cdot 2^2\cdot 2\cdot 2\cdot 3 \quad \boxed{a_z = 184}$$

$$\boxed{\bar{a}(6,3,2) = (18538, 0, 184)}$$

b) Diferencia de presiones entre (1) y (2):

$$\boxed{\cancel{\Delta P_{12} = \Phi_{(1)} - \Phi_{(2)}}}$$

Según Walter

$$\boxed{\bar{\nabla} p = \delta\,(\bar{g} - \bar{a}) + \mu \cdot \bar{\nabla}^2 \cdot \bar{v}} \quad (1)$$

$$\bar{\nabla} \cdot \bar{v} = \frac{\partial(16x^2 + y)}{\partial x} + \frac{\partial(10)}{\partial y} + \frac{\partial(y z^2)}{\partial z} = 32x + 2yz$$

$$\bar{\nabla}^2 \cdot \bar{v} = (32x + 2yz) \cdot \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = (32,\ 2z,\ 2y)$$

$$\boxed{\mu \bar{\nabla}^2 \cdot \bar{v} = \mu(32, 2z, 2y)}$$

$$\boxed{\bar{g} = -10\hat{j}}$$

- $a_x = 512x^3 + 32xy + 10$
- $a_y = 0$
- $a_z = 10z^2 + 2y^2 z^3$

$$\boxed{\bar{g} - \bar{a} = (-512x^3 - 32xy - 10, -10, -10z^2 - 2y^2 z^3)}$$

queda en (1); $\bar{\nabla} p = \left(-\delta(512x^3 + 32xy + 10) + 32\mu,\ -10\delta + \mu 2z,\ -\delta(10z^2 + 2y^2 z^3) + \mu 2y\right)$

tenemos $\bar{B} = \bar{\nabla} p$;

$$\boxed{\bar{B} = (B_x, B_y, B_z) = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right)} \quad (2)$$

integrando obtenemos p.

$$\frac{dp}{dx} = B_x \Rightarrow \qquad p = \int B_x \, dx$$

$$p = -\delta \frac{512}{4} x^4 - \delta \frac{32}{2} x^2 y - \delta 10 x + 32\mu x + g(y, z)$$

- $\frac{dp}{dy} = B_y$; $\quad \frac{dp}{dy} = -\delta 16x^2 + 32\mu + g_y$

$$\frac{dp}{dy} = -10\delta + \mu \cdot 2z$$

ver

c) $V: \mathbb{R} \longrightarrow \mathbb{R}$; $V(x,y,\tau) = \left(\frac{\;}{1+\tau}, \frac{\;}{1+2\tau}, \right)$

Hallar lines de corriente por (x_0, y_0, z_0) $\forall \tau$.

Tenemos, ~~$\bar{r}: \tau\in\mathbb{R}\to\mathbb{R}^3$~~ $\bar{r}: \tau \in \mathbb{R} \longrightarrow \mathbb{R}^3$

$\cancel{\frac{d\bar{r}}{d\tau} = \frac{dy}{d\tau}}$ $\quad \bar{V} = \frac{d\bar{r}}{d\tau} = \left(\frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right)$; a $\mathbb{R}^2$

$$\bar{V} = \left(\frac{dx}{d\tau} ; \frac{dy}{d\tau}\right) = (u, v) \text{ , queda}$$

$$\left.\begin{array}{l} u = \frac{dx}{d\tau} \Rightarrow , \quad \frac{dx}{u} = d\tau \\ v = \frac{dy}{d\tau} \Rightarrow \quad \frac{dy}{v} = d\tau \end{array}\right\} \Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v}}$$

resolvemos integrando;

$$(1+\tau)\int \frac{dx}{x} = (1+2\tau)\int\frac{dy}{y}$$

$$(1+\tau)\left[\ln|x| + \ln|c|\right] = (1+2\tau)\cdot\ln|y|$$

$$(1+\tau)\cdot\left[\ln|x\cdot c|\right] = (1+2\tau)\cdot\ln|y| \quad , \quad \log a^b = b\cdot\log a$$

$$e^{\ln\left[|x\cdot c|^{(1+\tau)}\right]} = e^{\ln\left(|y|^{1+2\tau}\right)}$$

$$|x\cdot c|^{(1+\tau)} = |y|^{(1+2\tau)}$$

$$|y| = |x\cdot c|^{\frac{1+\tau}{1+2\tau}} \quad , \quad \text{con } c_1 = c^{\frac{1+\tau}{1+2\tau}}$$

$$\boxed{|y| = c_1\cdot|x|^{1+\tau/1+2\tau}}$$

Para un punto (x_0, y_0), reemplazamos,

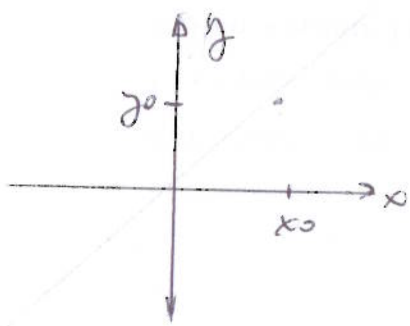
$|y_0| = C_1 \cdot |x_0| \Rightarrow \boxed{C_1 = \frac{|y_0|}{|x_0|^{(1+\tau)/(1+2\tau)}}}$ (6)

Finalmente:

$$\boxed{|y| = \frac{|y_0|}{|x_0|^{\frac{1+\tau}{1+2\tau}}} \, |x|^{1+\tau/1+2\tau}}$$

Para $\tau = 0$ $\quad \lim_{\tau \to 0} \frac{1+\tau}{1+2\tau} = \frac{1+0}{1+2\cdot 0} = 1$; queda:

$$\boxed{|y| = \frac{|y_0|}{|x_0|} \cdot |x|}$$ → rectas de pendiente $\frac{y_0}{x_0}$ pasantes por el origen ;



Para $\tau \to \infty$ $\quad \lim_{\tau \to \infty} \frac{1+\tau}{1+2\tau} = \lim_{\tau \to \infty} \frac{\overbrace{1/\tau}^{\to 0} + \tau/\tau}{\underbrace{1/\tau}_{\to 0} + 2\tau/\tau} = \frac{1}{2}$; queda:

$$|y| = \left| \frac{y_0}{x_0^{1/2}} \right| \cdot |x|^{1/2}$$ → curvas parabólicas de tipo $x = k y^2$