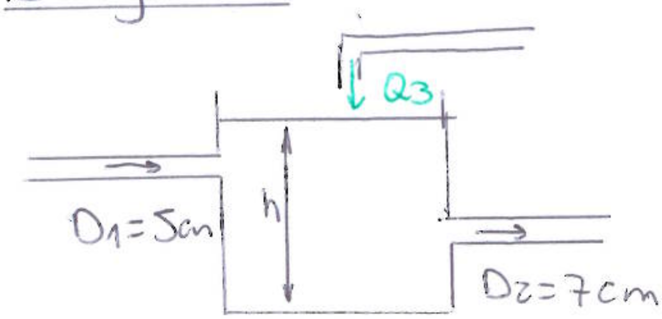
Teoría

Ej. 1 Hallar dh/dT

Diagrama



Datos

) Agua a 20°C, fluido incompresible
) $Q_3 = 0{,}01\ m^3/s$
) $D_1 = 5 \cdot 10^{-2}\ m$
) $D_2 = 7 \cdot 10^{-2}\ m$
) $\forall$: volumen,) v: velocidad

) Podemos encarar el ejercicio considerando el balance de masas mediante la ecuación de Reynolds. Tomamos como volumen control el recipiente;

$$B = m;\quad \beta = \frac{dB}{dm} = \frac{dm}{dm} = 1$$

$$\boxed{\frac{dB}{dT} = \frac{d}{dT}\left[\iiint_{Vc} \beta \cdot \rho \cdot d\forall\right] + \oiint_{Sc} \beta\, \rho \cdot \bar{v} \cdot \bar{n} \cdot dA} \quad (1)$$

En este caso, el primer término se anula, el Vc. no varía con el tiempo.

$$\frac{dm}{dT} = \oiint_{Sc} \rho \cdot \bar{v} \cdot \bar{n}\, dA$$ con $\rho = cte$; $v = cte$ para las secciones 1, 2, 3; $A = cte$,

$$\frac{dm}{dT} = \oiint_{Sc} \rho \cdot \bar{v} \cdot \bar{n} \cdot dA = \sum \rho \cdot v \cdot A$$; donde $\dot{m} = \rho \cdot v \cdot A$

$$\frac{dm}{dT} = \dot{m}_1 + \dot{m}_3 - \dot{m}_2$$; Aquí; $\frac{dm}{dT} \neq 0$

$$\boxed{\dot{m}_1 + \dot{m}_3 - \dot{m}_2 \neq 0} \quad (2)$$, requerimos $\frac{\Delta m}{\Delta T}$

$\Delta m = m_f - m_i$

$m = \rho \cdot V$, $\boxed{\dot{m} = \rho \cdot \dot{V}}$; en el Tanque; $V = A \cdot h$

$A = \frac{\pi \cdot D^2}{4}$, $\boxed{\dot{m} = \rho \cdot \frac{\pi \cdot D^2}{4} \cdot \frac{dh}{d\tau}}$ (3)

ver!

Problema 2

Hallar $\tau = f(g, d, \rho_a, \rho, \mu)$; Hallar expresión;

vemos las unidades en MTL:

	τ	g	d	ρ_a	ρ	μ
MTL	$\frac{M}{T^2 L}$	$\frac{L}{T^2}$	L	$\frac{M}{L^3}$	$\frac{M}{L^3}$	$\frac{M}{T \cdot L}$

Variables: 6

Unidades: 3

nros pi = 6 − 3 = 3

$$\boxed{\Pi_1 = \frac{\rho}{\rho_a}} \quad (1) \rightarrow \text{mismas unidades}$$

$$\boxed{\Pi_2 = \rho \cdot g^a \cdot d^b \cdot \mu^c}$$

$$M^0 L^0 T^0 = \left[\frac{M}{L^3}\right] \cdot \left[\frac{L}{T^2}\right]^a \cdot L^b \cdot \left[\frac{M}{L \cdot T}\right]^c$$

$$\left.\begin{array}{l} M^0) \Rightarrow 0 = 1 + c \\ T^0) \Rightarrow 0 = -2a - c \\ L^0) \Rightarrow 0 = -3 + a + b - c \end{array}\right\} \Rightarrow$$

~~c = −1; a = −c, a = 1; 0 = −3 + 1 + b − (−1); 0 = −3 + 1 + b + 1; 0 = −1 + b; b = 1~~

~~$\Pi_2 = \rho g d$~~

$\boxed{c = -1}$ $\quad 2a = -c; \quad a = -\frac{c}{2}; \quad \boxed{a = \frac{1}{2}}$

$$0 = -3 + \frac{1}{2} + b - (-1);$$

$$0 = -3 + \frac{1}{2} + b + 1$$

$$0 = -2 + \frac{1}{2} + b \quad ; \quad \boxed{b = \frac{3}{2}} \quad \text{; queda:}$$

$$\boxed{\Pi_2 = \frac{\rho \cdot g^{1/2} \cdot d^{3/2}}{\mu}}$$

$\Pi_3 = \tau \cdot \rho \cdot g \cdot d$

$$M_0 T^0 L^0 = \frac{M}{T^2 \cdot L} \cdot \left[\frac{M}{L^3}\right]^a \cdot \left[\frac{L}{T^2}\right]^b \cdot L^c$$

$$\left.\begin{array}{l} M^0) \Rightarrow 0 = 1 + a \\ T^0) \Rightarrow 0 = -2 - 2b \\ L^0) \Rightarrow 0 = -1 - 3a + b + c \end{array}\right\} \Rightarrow$$

$\boxed{a = -1}$

$2b = -2;\ \boxed{b = -1}$

$0 = -1 - 3(-1) - 1 + c$

$0 = -1 + 3 - 1 + c$

$\boxed{c = -1}$

$$\boxed{\Pi_3 = \tau \cdot \frac{1}{\rho} \cdot \frac{1}{g} \cdot \frac{1}{d}}$$

Agrego Π_2, Π_3.

$$\boxed{\frac{\tau}{\rho \cdot g \cdot d} = f\left(\frac{\rho \cdot g^{1/2} \cdot d^{3/2}}{\mu} ; \frac{\rho}{\rho_a}\right)}$$

Problema 3

a) Hallar $a(L)$

$$\bar{a} = \frac{D\bar{V}}{Dt} = \cancel{\frac{\partial \bar{V}}{\partial t}} + u \frac{\partial u}{\partial x} + \cancel{v \cdot \frac{\partial u}{\partial y}}$$

$$\bar{a} = V_0 \left(1 + \frac{2x}{L}\right) \cdot V_0 \cdot \frac{2}{L} \Rightarrow \boxed{\bar{a} = \frac{2 V_0^2}{L}\left(1 + \frac{2x}{L}\right)} \ (1)$$

b) $$\bar{a}(L) = 2 \cdot \frac{V_0^2}{L}\left(1 + \frac{2 \cdot L}{L}\right) \Rightarrow \frac{2 \cdot V_0^2}{L} \cdot (1+2);\ \boxed{\bar{a}(L) = \frac{6 V_0^2}{L}}$$

b) $$\bar{V} = \frac{dx}{dt} \qquad \int_0^t dt = \int_0^L \frac{dx}{V}$$

$$t - 0 = \int_0^L \frac{dx}{V_0\left(1 + \frac{2x}{L}\right)} = \frac{1}{V_0} \int_0^L \frac{dx}{\left(1 + \frac{2x}{L}\right)} = \frac{1}{V_0} \int_0^L \frac{dx}{\frac{L + 2x}{L}}$$

$t = \frac{L}{\ } \int^L \frac{dx}{\ } \qquad u = 2x + L \qquad dx = du$

$$t = \frac{\quad}{v_0}\int_0 \frac{\quad}{2u} \Rightarrow t = \frac{\quad}{v_0}\cdot\frac{\quad}{2}\int_0 \frac{\quad}{u}$$

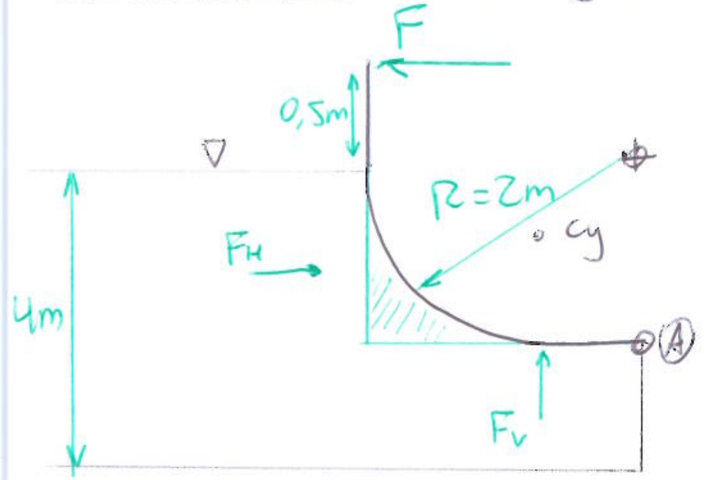
$$t = \frac{L}{2v_0}\cdot \ln|u|\Big]_0^L \Rightarrow t = \frac{L}{2v_0}\ln|2x+L|\Big]_0^L \Rightarrow$$

$$t = \frac{L}{2v_0}\cdot \ln\left(\frac{2\cdot L + L}{2\cdot 0 + L}\right) \Rightarrow t = \frac{L}{2\cdot v_0}\cdot \ln\left(\frac{3\cancel{L}}{\cancel{L}}\right)$$

$$\boxed{t = \ln(3)\cdot\frac{L}{2v_0}}$$

Practica

Ejercicio 1 Diagrama



Hallar F para mantener compuerta en equilibrio

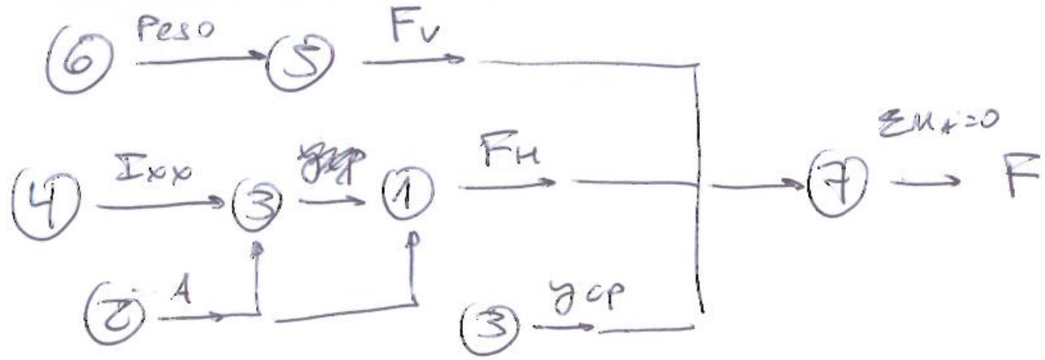
) Calculamos las Fuerzas horizontal, vertical y el centro de presión. Luego se plantea ec. de momentos para el equilibrio de la compuerta

$\cancel{F_H = \gamma_{H_2O}\cdot g\cdot h_{cg}}$ $\cancel{F_H = \gamma g}$

$$\boxed{F_H = \gamma_{H_2O}\cdot h_{cg}\cdot Area_H} \quad (1) \qquad \boxed{Area_H = b\cdot h} \quad (2)$$

$$\boxed{y_{cp} = y_{cg} + \frac{I_{xx}}{y_{cg}\cdot Area_H}} \quad (3) \qquad \boxed{I_{xx} = \frac{b\cdot h^3}{12}} \quad (4)$$

$$\boxed{F_V = \gamma_{H_2O}\cdot h_A\cdot Area_V - [peso \ \square - peso \ \square]} \quad (5)$$



De (6) en (5)

$$F_V = \gamma \cdot h_A \cdot R \cdot b - \gamma \cdot R^2 \cdot b + \gamma \cdot \frac{\pi \cdot R^2}{4} \cdot b$$

$$F_V = \gamma \cdot R \cdot b \left[h_A - R + \frac{\pi}{4} \cdot R \right]$$

$$F_V = 10^3 \frac{kg}{\cancel{m^3}} \cdot 9{,}81 \frac{m}{s^2} \cdot 2\cancel{m} \cdot 4\cancel{m} \cdot \left[2 - 2 + \frac{\pi}{4} \cdot 2 \right] \cancel{m}$$

$$\boxed{F_V = 123276\ N} \quad (6)$$

De (4) en (3);

$$y_{cp} = y_{cg} + \frac{\cancel{b} \cdot h^{\cancel{3}\,2}}{12 \cdot y_{cg} \cdot \cancel{b} \cdot \cancel{h}} \quad \text{Ayuda: } h = R \quad ; \quad y_{cp} = y_{cg} + \frac{R^2}{12 \cdot y_{cg}}$$

con $y_{cg} = 1\ m$;

$$y_{cp} = 1m + \frac{2^2\, m^{\cancel{2}}}{12 \cdot 1\cancel{m}} \quad ; \quad \boxed{y_{cp} = \frac{4}{3}\ m} \ (3)$$

De (1);

$$F_H = 10^3 \frac{kg}{\cancel{m^3}} \cdot 9{,}81 \frac{m}{s^2} \cdot 1\cancel{m} \cdot 2\cancel{m} \cdot 4\cancel{m}$$

$$\boxed{F_H = 78480\ N} \ (1) \quad ; \quad \text{planteamos} \ \boxed{\Sigma M_A = 0}$$

$$\Sigma M_A = F \cdot 2{,}5m - F_H \cdot (2m - y_{cp}) - F_V \cdot \frac{4R}{3\pi} = 0$$

$$F \cdot 2{,}5m = F_H \left(2m - \frac{4}{3} m\right) + F_V \cdot \frac{4R}{3\pi} = 0$$

$$\cancel{F = \frac{123276\ N (2/3) m}{2{,}5m} +}$$

$$F = \frac{78480\ N \cdot \left(\frac{2}{3}\right) \cancel{m}}{2{,}5 \cancel{m}} + \frac{123276\ N \cdot 4 \cdot 2 \cancel{m}}{3\pi \cdot 2{,}5 \cancel{m}}$$

$$F = 20928\ N + 41855\ N \ ; \ \boxed{F = 62783\ N}$$

Ejercicio N° 2

1) Por continuidad:

$$P_A + \cancel{\gamma_{H_2O} \cdot a} + \gamma_{Hg} \cdot 2a - \cancel{\gamma_{H_2O} \cdot a} = P_B$$

$$P_{BA} = \gamma_{Hg} \cdot 2a \Rightarrow \boxed{a = \frac{P_{BA}}{2 \gamma_{Hg}}}$$

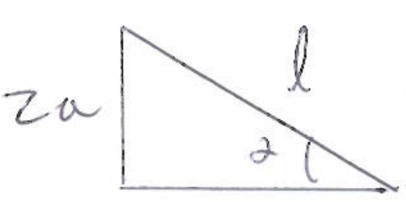
$$a = \frac{20 \cdot 10^3\ Pa \cdot m^3 \cdot s^2}{2 \cdot 13560\ kg \cdot 9{,}81\ m}$$

~~unidades:~~ $\cancel{\frac{Pa \cdot m^3}{kg} = \frac{N}{m^2} \frac{m^3}{kg} = \frac{kg \cdot m}{s^2} \frac{m}{kg}}$

unidades: $\frac{Pa \cdot m^{\cancel{3}\,2} \cdot s^2}{kg \cdot \cancel{m}} = \frac{N}{\cancel{m^2}} \frac{\cancel{m^2} \cdot s^2}{kg} = \frac{\cancel{kg} \cdot m}{\cancel{s^2}} \frac{\cancel{s^2}}{\cancel{kg}}$

$$\boxed{a = 0{,}075\ m}$$

2) ¿ángulo α? por geometría del esquema:



$$\sin(\alpha) = \frac{2a}{l}$$

$$\alpha = a\sin\left(\frac{2a}{l}\right) = a\sin\left(\frac{2 \cdot 0{,}075\ m}{0{,}268\ m}\right)$$

Ejercicio

Realizamos diagrama de cuerpo libre para aguja; no considero empuje.

F_T: Fuerza por tensión superficial

W: peso del cuerpo.

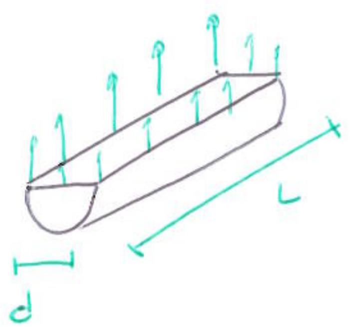
$$\boxed{-F_T + W = \Sigma F = 0} \quad (1)$$

$$\boxed{W = \gamma_{Fe} \cdot Vol} \quad (2) \qquad \boxed{Vol = L \cdot Área = L \frac{\pi d^2}{4}} \quad (3)$$

$$\boxed{\gamma_{Fe} = \rho_{Fe} \cdot g = S_{g\,Fe} \cdot \gamma_{H_2O}} \quad (4)$$

S_g: gravedad específica

Para la tensión superficial (γ)



$$\gamma = \frac{Fuerza}{longitud} \; ; \qquad \gamma = \frac{F_T}{\Sigma \ell} \; ;$$

$$F_T = \gamma \cdot \Sigma \ell$$

$$F_T = \gamma \cdot (2L + 2d)$$

$$\boxed{F_T = \gamma \cdot 2(L + d)} \quad (5)$$

Esquema:

$$\begin{tikzcd}
(4) \arrow[r, "\gamma_{Fe}"] & (2) \arrow[r, "W"] & (1) \arrow[r] & (D) \\
(3) \arrow[ur, "vol"] & (5) \arrow[ur, "F_T"] & &
\end{tikzcd}$$

$$\left.\begin{matrix}(4)\\(3)\end{matrix}\right\} \rightarrow (2); \qquad W = \gamma_{Fe} \cdot Vol \; ; \quad (6) \; \boxed{W = S_{g\,Fe} \cdot \gamma_{H_2O} \cdot L \cdot \frac{\pi \cdot d^2}{4}}$$

$$\left.\begin{matrix}(5)\\(6)\end{matrix}\right\} \rightarrow (1) \qquad -\gamma \cdot 2(L + d) + S_{g\,Fe} \cdot \gamma_{H_2O} \cdot L \cdot \frac{\pi \cdot d^2}{4} = 0$$

$Sg\,Fe \cdot \gamma_{H2O} \cdot L \cdot \frac{\pi}{4} \cdot d^2 = 2\gamma(L+d)$

a) Si despreciamos acción de F_T sobre diámetro;

$$d^2 = \frac{8 \cdot \gamma \cdot (L+d)}{Sg\,Fe \cdot \gamma_{H2O} \cdot L \cdot \pi} \; ; \quad d \to 0$$

$$d = \sqrt{\frac{8 \cdot \gamma \cdot \cancel{L}}{Sg\,Fe \cdot \gamma_{H2O} \cdot \cancel{L} \cdot \pi}} \; , \quad \boxed{d = \sqrt{\frac{8\gamma}{Sg\,Fe \cdot \gamma_{H2O} \cdot \pi}}}$$

b) Sin despreciar acción de F_T sobre el diámetro.

$$Sg\,Fe \cdot \gamma_{H2O} \cdot L \cdot \frac{\pi}{4} \cdot d^2 = 2\gamma \cdot L + 2\gamma \cdot d$$

$$\boxed{Sg\,Fe \cdot \gamma_{H2O} \cdot L \cdot \frac{\pi}{4} \cdot d^2 - 2\gamma \cdot d - 2\gamma L = 0}$$

queda $a \cdot d^2 + b \cdot d + c = 0$; con

$$d_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$