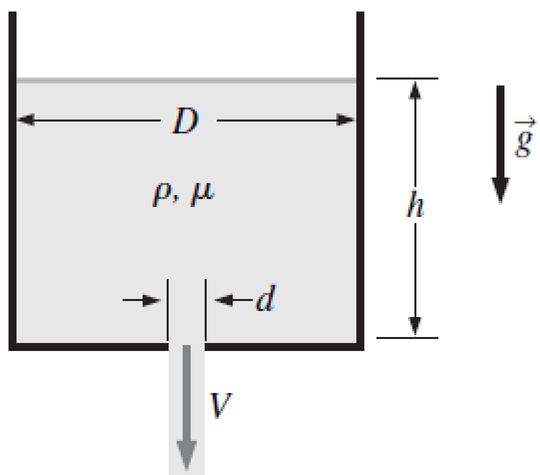
Modelo Segundo Parcial

Ejercicio 1

Un líquido de densidad ρ y viscosidad μ fluye por gravedad a través de un agujero de diámetro d en el fondo de un tanque de diámetro D (Fig. P7-90). Al principio del experimento, la superficie del líquido está a la altura h sobre el fondo del tanque, como se muestra. El líquido sale del tanque como un chorro con velocidad promedio V directo hacia abajo, como también se muestra. Con análisis dimensional genere una relación adimensional para V como función de los otros parámetros en el problema. Identifique cualquier parámetro adimensional establecido (nombrado) que aparezca en su resultado. (Sugerencia: en este problema existen tres longitudes. Por coherencia, elija h como su longitud característica.)



Solution We are to find the functional relationship between the given parameters and name any established dimensionless parameters.

Assumptions **1** The given parameters are the only ones relevant to the flow at hand.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are seven parameters in this problem; $n = 7$,

List of relevant parameters: $$V = f(d, D, \rho, \mu, h, g) \qquad n = 7 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

V	d	D	ρ	μ	h	g
$\{L^1t^{-1}\}$	$\{L^1\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1\}$	$\{L^1t^{-2}\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $$k = n - j = 7 - 3 = 4$$

Step 4 We need to choose three repeating parameters since $j = 3$. We pick length scale h, fluid density ρ, and gravitational constant g.

Repeating parameters: h, ρ, and g

Step 5 The Πs are generated. Note that in this case we do the algebra in our heads since these relationships are very simple. The dependent Π is

Π_1 = *a Froude number:* $$\Pi_1 = \frac{V}{\sqrt{gh}}$$

This Π is a type of **Froude number**. Similarly, the two length-scale Πs are obtained easily,

Π_2: $$\Pi_2 = \frac{d}{h}$$

and

Π_3: $$\Pi_3 = \frac{D}{h}$$

Finally, the Π formed with viscosity is generated,

$$\Pi_4 = \mu h^{a_4} \rho^{b_4} g^{c_4} \qquad \{\Pi_4\} = \left\{(m^1L^{-1}t^{-1})(L^1)^{a_4}(m^1L^{-3})^{b_4}(L^1t^{-2})^{c_4}\right\}$$

mass: $\{m^0\} = \{m^1 m^{b_4}\}$ $\qquad 0 = 1 + b_4 \qquad b_4 = -1$

time: $\{t^0\} = \{t^{-1}t^{-2c_4}\}$ $\qquad 0 = -1 - 2c_4 \qquad c_4 = -\frac{1}{2}$

length: $\{L^0\} = \{L^{-1}L^{a_4}L^{-3b_4}L^{c_4}\}$ $\quad 0 = -1 + a_4 - 3b_4 + c_4$ $\quad$ $a_4 = -\dfrac{3}{2}$

$$0 = -1 + a_4 + 3 - \tfrac{1}{2}$$

which yields

Π_4:

$$\Pi_4 = \frac{\mu}{\rho h^{\frac{3}{2}}\sqrt{g}}$$

We recognize this Π as the inverse of a kind of **Reynolds number**. We also split the h terms to separate them into a length scale and (when combined with g) a velocity scale. The final form is

Modified Π_4 = *a Reynolds number:*

$$\Pi_4 = \frac{\rho h\sqrt{gh}}{\mu}$$

Step 6 We write the final functional relationship as

Relationship between Π*s:*

$$\boxed{\frac{V}{\sqrt{gh}} = f\left(\frac{d}{h}, \frac{D}{h}, \frac{\rho h\sqrt{gh}}{\mu}\right)} \qquad (2)$$

Discussion You may choose different repeating variables, and may generate different nondimensional groups. If you do the algebra correctly, your answer is not "wrong" – you just may not get the same dimensionless groups.

Ejercicio 2

Un rotor de helicóptero con cuatro palas gira a *n* rpm en aire de propiedades (ρ, μ). Cada pala tiene una cuerda *C* y se extiende desde el eje de rotación hasta una distancia *R* (se desprecia el tamaño de la cabeza del rotor). Suponiendo flujo turbulento desde el borde de ataque, desarrolle una estimación analítica de la potencia *P* necesaria para mover dicho rotor.

Solution: The "freestream" velocity varies linearly from root to tip, as shown in the figure. Thus the drag force on a strip (C dr) of blade is, for turbulent flow,

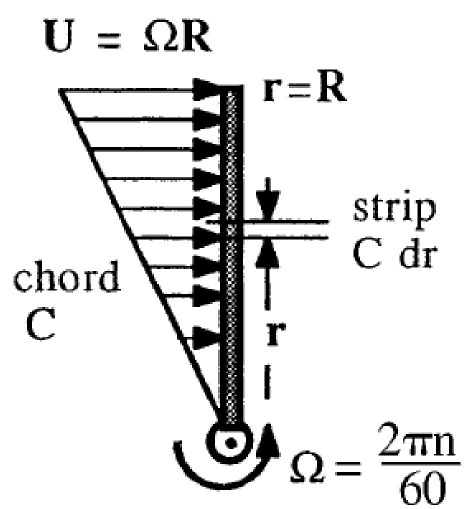


Fig. P7.42

$$dF = \frac{0.031}{Re_C^{1/7}}\left(\frac{\rho}{2}\right)u^2 C\,dr\ (2\text{ sides}) \approx 0.031\mu^{1/7}\rho^{6/7}C^{6/7}(\Omega r)^{13/7}dr, \quad \text{where } u = \Omega r.$$

$$\text{or}\quad \text{Power} = \int_{\text{blade}} u\,dF\ (4\text{ blades}) = 4(0.031)\mu^{1/7}\rho^{6/7}C^{6/7}\Omega^{20/7}\int_0^R r^{20/7}dr$$

Finally, after cleaning up, $P_{4\text{ blades}} \approx \mathbf{0.0321\mu^{1/7}\rho^{6/7}C^{6/7}\Omega^{20/7}R^{27/7}}$ *Ans.*

Ejercicio 3

8-137 Un líquido sumamente viscoso se descarga de un contenedor grande a través de un tubo de diámetro pequeño en flujo

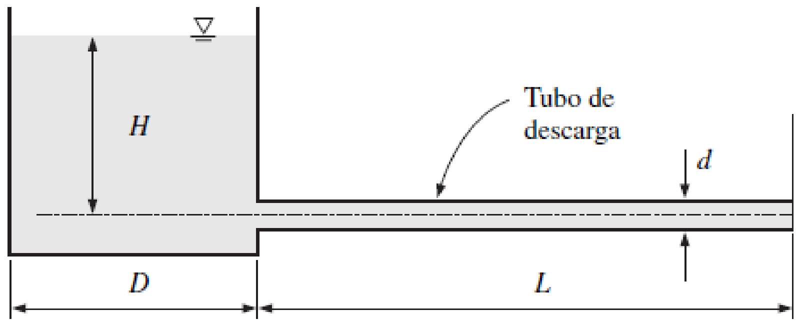
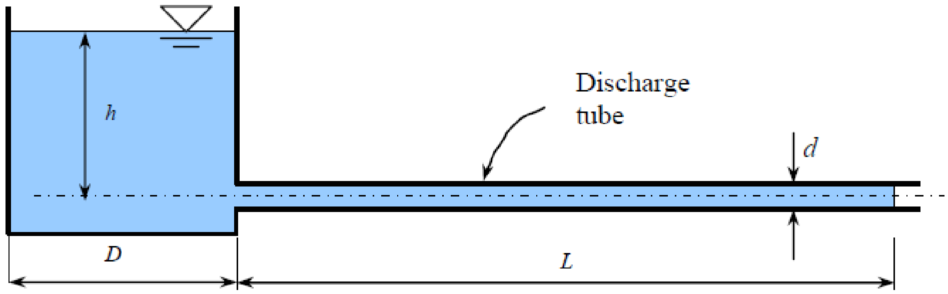


FIGURA P8-137

laminar. Sin considerar efectos de entrada y cargas de velocidad, obtenga una relación para la variación de la profundidad del fluido en el tanque con el tiempo.

Solution A highly viscous liquid discharges from a large container through a small diameter tube in laminar flow. A relation is to be obtained for the variation of fluid depth in the tank with time.



Assumptions **1** The fluid is incompressible. **2** The discharge tube is horizontal, and the flow is laminar. **3** Entrance effects and the velocity heads are negligible.

Analysis We take point 1 at the free surface of water in the tank, and point 2 at the exit of the pipe. We take the centerline of the pipe as the reference level ($z_1 = h$ and $z_2 = 0$). Noting that the fluid at both points 1 and 2 are open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low $(V_1 \cong 0)$ and the velocity heads are negligible, the energy equation for a control volume between these two points gives

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \quad \rightarrow \quad \frac{P_{atm}}{\rho g} + h = \frac{P_{atm}}{\rho g} + h_L \quad \rightarrow \quad h_L = h \qquad (1)$$

where h is the liquid height in the tank at any time t. The total head loss through the pipe consists of major losses in the pipe since the minor losses are negligible. Also, the entrance effects are negligible and thus the friction factor for the entire tube is constant at the fully developed value. Noting that $f = \mathrm{Re}/64$ for fully developed laminar flow in a circular pipe of diameter d, the head loss can be expressed as

$$h_L = f \frac{L}{d} \frac{V^2}{2g} = \frac{64}{\mathrm{Re}} \frac{L}{d} \frac{V^2}{2g} = \frac{64}{Vd/\nu} \frac{L}{d} \frac{V^2}{2g} = \frac{64\nu L}{d^2} \frac{V}{2g} \qquad (2)$$

The average velocity can be expressed in terms of the flow rate as $V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi d^2/4}$. Substituting into (2),

$$h_L = \frac{64\nu L}{d^2} \frac{1}{2g} \left(\frac{\dot{V}}{\pi d^2/4} \right) = \frac{64\nu L}{d^2} \frac{4\dot{V}}{2g\pi d^2} = \frac{128\nu L \dot{V}}{g\pi d^4} \qquad (3)$$

Combining Eqs. (1) and (3): $$h = \frac{128\nu L \dot{V}}{g\pi d^4} \qquad (4)$$

Noting that the liquid height h in the tank decreases during flow, the flow rate can also be expressed in terms of the rate of change of liquid height in the tank as

$$\dot{V} = -A_{\text{tank}} \frac{dh}{dt} = -\frac{\pi D^2}{4} \frac{dh}{dt} \qquad (5)$$

Substituting Eq. (5) into (4): $$h = -\frac{128\nu L}{g\pi d^4} \frac{\pi D^2}{4} \frac{dh}{dt} = -\frac{32\nu L D^2}{g d^4} \frac{dh}{dt} \qquad (6)$$

To separate variables, it can be rearranged as $$dt = -\frac{32\nu L D^2}{g d^4} \frac{dh}{h}$$

Integrating from $t = 0$ (at which $h = H$) to $t = t$ (at which $h = h$) gives

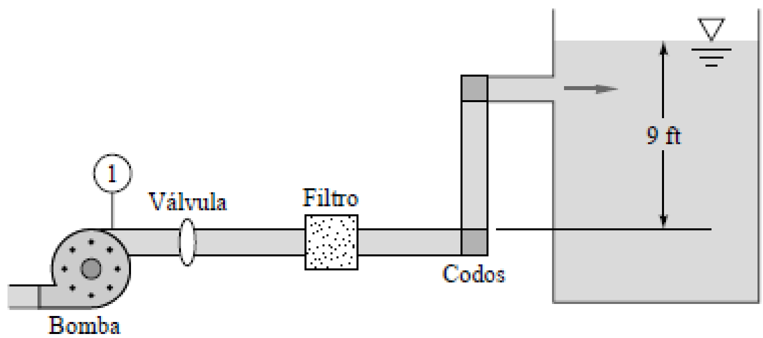
$$\boxed{t = \frac{32\nu L D^2}{g d^4} \ln(H/h)}$$

which is the desired relation for the variation of fluid depth h in the tank with time t.

Discussion If the entrance effects and the outlet kinetic energy were included in the analysis, the time would be slower.

Ejercicio 4

La bomba de agua de la Figura P6.108 mantiene una presión de 6,5 psi en el punto 1. A continuación hay una válvula de disco medio abierta, un filtro y dos codos roscados a 90°. La tubería comercial de acero tiene una longitud total de 80 ft. (*a*) Si el caudal es de 0,4 ft^3/s, ¿cuál es el coeficiente de pérdida del filtro? (*b*) Si la válvula de disco está completamente abierta y el coeficiente de pérdida del filtro es $K_{filtro} = 7$, ¿cuál es el caudal resultante?



P6.108

Solution: For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. The energy equation is written from point 1 to the surface of the tank:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f+K_{valve}+K_{filter}+2K_{elbow}+K_{exit}$$

(a) From the flow rate, V_1 = Q/A = (0.4 ft^3/s)/[(π/4)(4/12 ft)^2] = 4.58 ft/s. Look up minor losses and enter into the energy equation:

$$\frac{(6.5)(144)\text{ lbf/ft}^2}{62.4\text{ lbf/ft}^3}+\frac{(4.58\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}+0$$

$$=0+0+9\text{ ft}+\frac{(4.58)^2}{2(32.2)}\left[f\frac{80\text{ ft}}{(4/12\text{ ft})}+2.8+K_{filter}+2(0.64)+1\right]$$

We can solve for K_{filter} if we evaluate f. Compute Re_D = (1.94)(4.58)(4/12)/(2.09E−5) = 141,700. For commercial steel, ε/D = 0.00015 ft/0.333 ft = 0.00045. From the Moody chart, $f \approx 0.0193$, and $fL/D = 4.62$. The energy equation above becomes:

$$15.0\text{ ft}+0.326\text{ ft}=9\text{ ft}+0.326(4.62+2.8+K_{\text{filter}}+1.28+1)\text{ ft},$$

$$\text{Solve}\quad \boldsymbol{K_{\text{filter}}\approx 9.7}\quad \textit{Ans. (a)}$$

(b) If K_{filter} = 7.0 and V is unknown, we must iterate for the velocity and flow rate. The energy equation becomes, with the disk valve wide open ($K_{Valve} \approx 0$):

$$15.0\text{ ft}+\frac{V^2}{2(32.2)}=9\text{ ft}+\frac{V^2}{2(32.2)}\left(f\frac{80}{1/3}+0+7.0+1.28+1\right)$$

$$\textit{Iterate to find}\quad f\approx 0.0189,\quad Re_D=169{,}000,\quad V=5.49\text{ ft/s},$$

$$\mathbf{Q=AV=0.48\ ft^3/s}\quad \textit{Ans. (b)}$$

Ejercicio 5

Una turbina desarrolla 144 CV girando a 100 rpm bajo una carga de 8,0 m. (*a*) ¿Qué potencia desarrollaría bajo una carga de 11,0 m, suponiendo el mismo caudal? (*b*) ¿A qué velocidad giraría la turbina?

Solución:

(*a*) Potencia desarrollada = $wQHe/75$, de donde $wQe/75 = CV/H = 144/8$.

Para el mismo caudal (y rendimiento), bajo la carga de 11 m, obtenemos

$$wQe/75 = 144/8 = CV/11 \quad \text{o} \quad CV = 198$$

(*b*)

$$N_S = \frac{N\sqrt{P}}{H^{5/4}} = \frac{100\sqrt{144}}{(8)^{5/4}} = 89{,}19 \text{ rpm}$$

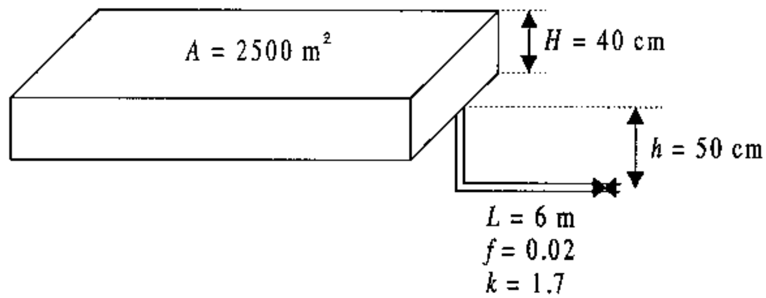
Luego

$$N = \frac{N_S H^{5/4}}{\sqrt{P}} = \frac{89{,}19(11)^{5/4}}{\sqrt{198}} = 127 \text{ rpm}$$

Ejercicio 6

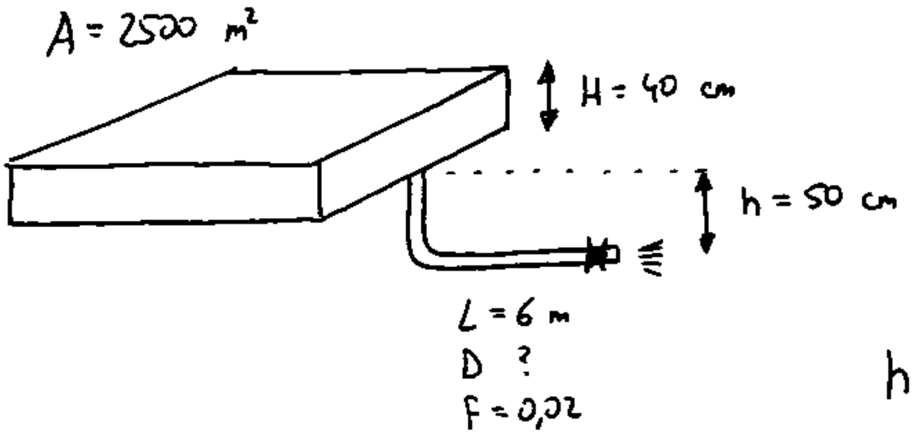
El vaciado de un estanque de sección $A = 2500\ m^2$ y altura $H = 40$ cm se realiza mediante una tubería de longitud $L = 6$ m y factor de fricción $f = 0.02$ en cuyo extremo hay una válvula de compuerta (el coeficiente de pérdidas menores de toda la instalación es $k = 1.7$). La sección de salida de la tubería de vaciado se encuentra a un desnivel $h = 50$ cm sobre la solera del estanque.

Despreciando el término de inercia, determinar el diámetro mínimo de la tubería para que el estanque se vacíe completamente durante un día.



Pérdidas por fricción = $f \dfrac{L}{D} \dfrac{v^2}{2g}$

Pérdidas menores = $k \dfrac{v^2}{2g}$



$$h_{locales} = k \cdot \frac{v^2}{2g}$$

SALIDA DEPÓSITO → $k = 0,5$

CODO 90° → $k = 0,9$

VALVULA COMPUERTA → $k = 0,3$

$1,7$

BERNOULLI

$$z(t) + h = \frac{v(t)^2}{2g} + f \frac{L}{D} \frac{v(t)^2}{2g} + k \frac{v(t)^2}{2g}$$

CONTINUIDAD

$$A \cdot \frac{dz(t)}{dt} = - v(t) \cdot \frac{\pi D^2}{4}$$

Tenemos 2 ecuaciones con 2 incógnitas → $z(t)$ y $v(t)$

$$z(t) + h = \left(1 + f\frac{L}{D} + k\right) \cdot \frac{v(t)^2}{2g}$$

$$v(t) = \sqrt{\frac{2g\left(z(t) + h\right)}{1 + f\frac{L}{D} + k}}$$

sustituyendo $v(t)$ en la ecuación de continuidad:

$$A \cdot \frac{dz(t)}{dt} = -\frac{\pi D^2}{4}\sqrt{\frac{2g}{1+f\frac{L}{D}+k}} \cdot \sqrt{z(t)+h}$$

ecuación diferencial de variables separables:

$$\int_H^0 \frac{dz}{\sqrt{z+h}} = -\frac{\pi D^2}{4A}\sqrt{\frac{2g}{1+f\frac{L}{D}+k}} \int_0^T dt$$

$$2\cdot\sqrt{z+h}\,\Big|_{z=H}^{z=0} = -\frac{\pi D^2}{4A}\cdot\sqrt{\frac{2g}{1+f\frac{L}{D}+k}}\cdot t\,\Big|_{t=0}^{t=T}$$

$$\boxed{2\sqrt{h} - 2\sqrt{H+h} = -\frac{\pi D^2}{4A}\cdot\sqrt{\frac{2g}{1+f\frac{L}{D}+k}}\cdot T}$$

sustituyendo valores:

$$-0{,}48315 = -3{,}14159 \cdot 10^{-4} \cdot D^2 \cdot \sqrt{\frac{19{,}62}{2{,}7 + \frac{0{,}12}{D}}} \cdot 86400$$

$$D^2 = 0{,}0178 \cdot \sqrt{\frac{2{,}7 + \frac{0{,}12}{D}}{19{,}62}}$$

iterando:

$$D = 0{,}1 \text{ m} \longrightarrow 0{,}089 \text{ m} \longrightarrow 0{,}0899 \text{ m}$$

$$\boxed{D = 90 \text{ mm}}$$