P1

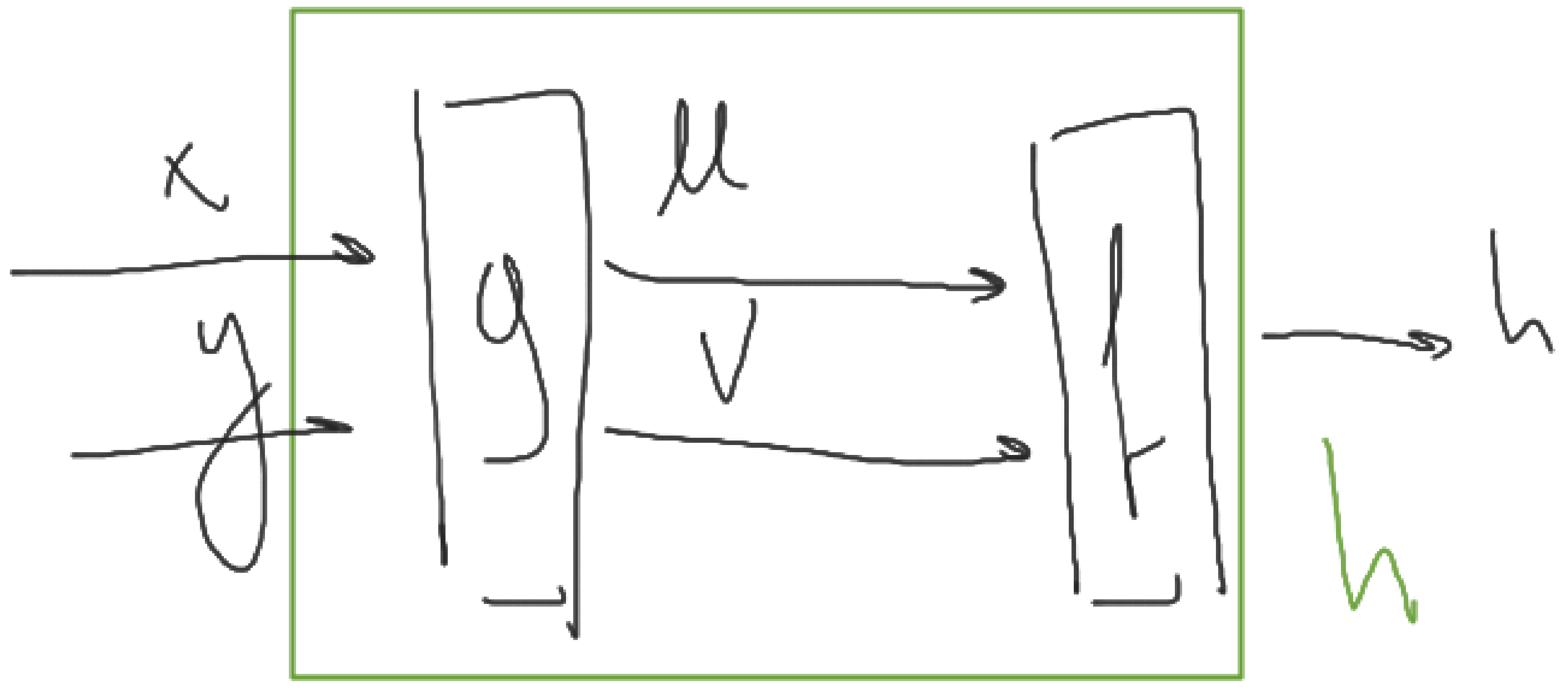
Sea $h = f \circ \vec{g}$ con $\vec{g}(x,y) = (\overbrace{-x+2y}^{u}, \overbrace{x-2y}^{v})$ y $f : \mathbb{R}^2 \to \mathbb{R}$ tal que $f \in C^1(\mathbb{R}^2)$. Entonces, la ecuación $2\,h'_x(x,y) + h'_y(x,y) = 0$
se cumple para:

1 f

$h = \overset{2}{f} \circ \overset{1}{g}$

1) $g : \mathbb{R}^2 \to \mathbb{R}^2$

2) $f : \mathbb{R}^2 \to \mathbb{R}$



(11) **Regla de la cadena.**
Sean $\mathbf{f}: A \subset \mathbb{R}^n \to \mathbb{R}^m$ y $\mathbf{g}: B \subset \mathbb{R}^m \to \mathbb{R}^k$ dos funciones tales que $\mathbf{f}(A) \subset B$, A y B abiertos.
Si $\mathbf{f}$ es diferenciable en $\mathbf{x}_0 \in A$ y $\mathbf{g}$ es diferenciable en $\mathbf{f}(\mathbf{x}_0)$, entonces $\mathbf{g} \circ \mathbf{f}$ es diferenciable en $\mathbf{x}_0$ y $D(\mathbf{g} \circ \mathbf{f})(\mathbf{x}_0) = D\mathbf{g}(\mathbf{f}(\mathbf{x}_0))D\mathbf{f}(\mathbf{x}_0)$.

$h = f \circ g$,

$$\boxed{\vec{\nabla} h = (h_x, h_y)_{(x_0, y_0)} = Df|_{(u_0, v_0)} \cdot Dg(x_0, y_0)}$$

$$\vec{g}(x,y) = (\underbrace{-x+2y}_{u},\ \underbrace{x-2y}_{v})$$

$$f : \mathbb{R}^2 \to \mathbb{R}$$

$$\boxed{Dg = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}}$$

$$\boxed{Df|_{(u_0, v_0)} = (f_u, f_v)}$$

$$(h_x, h_y) = (f_u, f_v) \cdot \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}$$

$$(h_x, h_y) = (l_u, l_v)\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} = (-l_u + l_v,\ 2l_u - 2l_v)$$

Entonces, la ecuación $2\,h'_x(x,y) + h'_y(x,y) = \alpha$

$$2\left[-l_u + l_v\right] + 2l_u - 2l_v = \alpha$$

$$\cancel{-2l_u} + \cancel{2l_v} + \cancel{2l_u} - \cancel{2l_v} = \alpha \ ; \quad \alpha = 0$$

P2

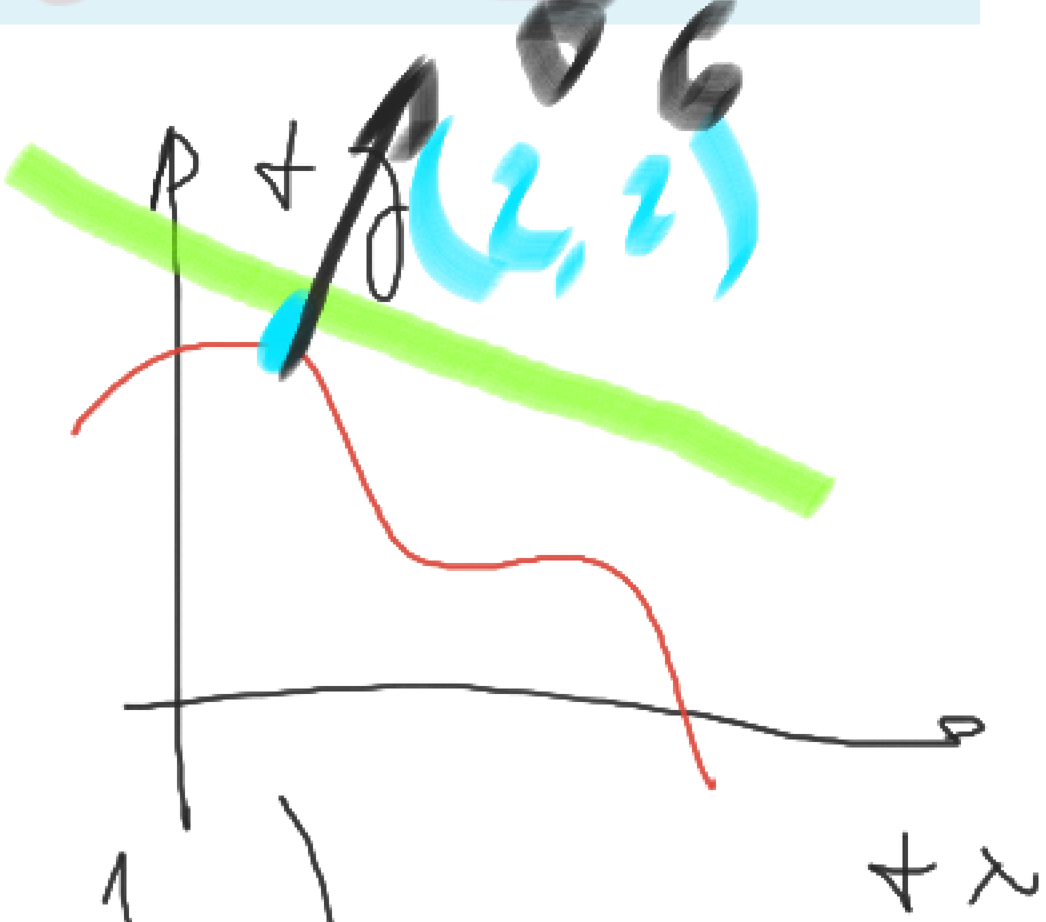
Sean C la curva plana de ecuación $2\,x\,y+\ln(x^2-y-1)-12\,x+16=0$ y f el campo escalar tal que $f(x,y)=x^2\,y-2\,y$. Entonces, la derivada direccional de f en $(1,4)$ en la dirección tangente a C el punto $(2,2)$ y orientada hacia y^+ es:

2

1

1

$$\underbrace{2\,x\,y+\ln(x^2-y-1)-12\,x+16=0}_{G(x,y)}$$



$$\bar{\nabla} G=(G_x,G_y)=\left(2y+\frac{2x}{x^2-y-1}-12;\ 2x-\frac{1}{x^2-y-1}\right)$$

$$\bar{\nabla} G(2,2)=(-4,3)$$

$$\bar{V}=(a,b),\ \bar{V}_{\perp}=(b,-a)$$

$\bar{\nabla} G(2,2) = (-4, 3)$

$V = (3,4)$ normal forb

$V = (-3,-4)$ $\|V\| = \sqrt{3^2+4^2} = 5$

$$\boxed{\hat{V} = \frac{\bar{V}}{\|V\|} = \frac{(3,4)}{5} = \left(\frac{3}{5}, \frac{4}{5}\right)}$$

$$f(x,y) = x^2y - 2y$$

$\bar{\nabla}f = (f_x, f_y) = (2xy, x^2-2)$

(14) **Proposición.** Si $f \colon A \subset \mathbb{R}^n \to \mathbb{R}$ es diferenciable en $\mathbf{x}_0$, entonces existe la derivada de f en $\mathbf{x}_0$ en cualquier dirección y $\frac{\partial f}{\partial \mathbf{v}}(\mathbf{x}_0) = \nabla f(\mathbf{x}_0) \cdot \mathbf{v}$.

$\bar{\nabla} f(1,4) = (8,-1)$

$$\bar{\nabla} f(1,4) = (8,-1) \qquad \hat{v} = \frac{\bar{v}}{\|v\|} = \frac{(3,4)}{5} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$\frac{\partial f}{\partial v}(1,4) = \bar{\nabla} f(1,4) \cdot \hat{v} = (8,-1)\cdot\left(\frac{3}{5}, \frac{4}{5}\right) = \frac{24-4}{5} = \frac{20}{5}$$

$$= 4$$

Seleccione una:

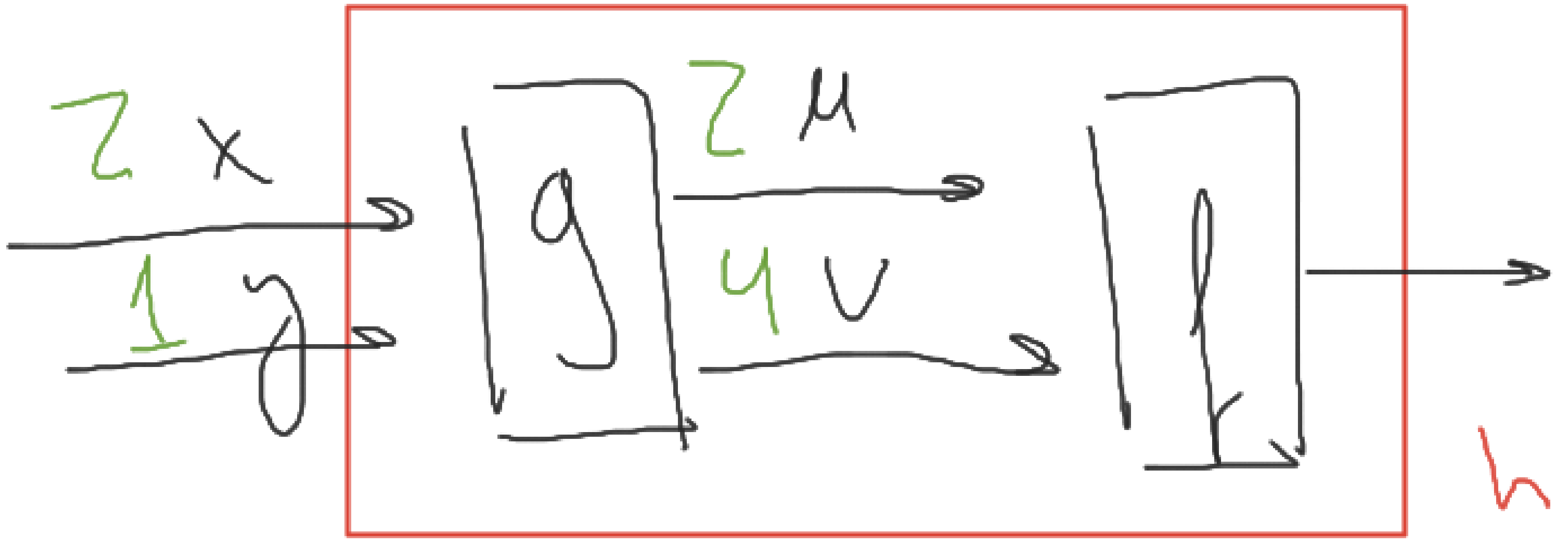
- a. Ninguna de las otras es correcta
- b. -29
- c. -20
- d. 4
- e. 0

P3

Sea el campo escalar $f \in C^1(\mathbb{R}^2)$ con $\nabla f(2,4) = (1,3)$ y $\vec{g}(x,y) = (\underbrace{x\,y}_{u},\ \underbrace{x^2}_{v})$. Entonces el valor de la máxima derivada direccional de $h = f \circ \vec{g}$ en el punto $\vec{A} = (2,1)$ es:

(15) **Proposición.** Con las hipótesis anteriores, si además $\nabla f(x_0) \neq \mathbf{0}$, entonces existe la derivada direccional máxima de f en $\mathbf{x}_0$, vale $\|\nabla f(\mathbf{x}_0)\|$ y se alcanza en la dirección $\mathbf{v} = \dfrac{\nabla f(\mathbf{x}_0)}{\|\nabla f(\mathbf{x}_0)\|}$.

$h = f \circ g$



$\|\nabla h\|$

$g(2,1) = (2,4)$

$\nabla h(2,1) = \nabla f(2,4) \cdot \nabla g(2,1)$

Regla de la cadena

$$\nabla h(2,1) = \nabla f(2,4) \cdot \nabla g(2,1) = (1,3) \cdot \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}_{(2,1)}$$

$$\vec{g}(x,y) = (\overbrace{x\,y}^{u}, \overbrace{x^2}^{v})$$

$$= (1,3) \cdot \begin{pmatrix} y & x \\ 2x & 0 \end{pmatrix}_{(2,1)}$$

$$\nabla h = (1,3) \begin{pmatrix} 1 & 2 \\ 4 & 0 \end{pmatrix}$$

$$\boxed{\nabla h = (13, 2)}$$

$\boxed{\bar{D}h = (13, 2)}$ $\quad \|\bar{D}h\| = \sqrt{13^2 + 6^2}$

Seleccione una:

- a. $h'(\vec{A}, \breve{r}_{max}) = \sqrt{169}$
- b. $h'(\vec{A}, \breve{r}_{max}) = \sqrt{411}$
- c. $h'(\vec{A}, \breve{r}_{max}) = \sqrt{173}$
- d. Ninguna de las otras es correcta
- e. $h'(\vec{A}, \breve{r}_{max}) = \sqrt{26}$

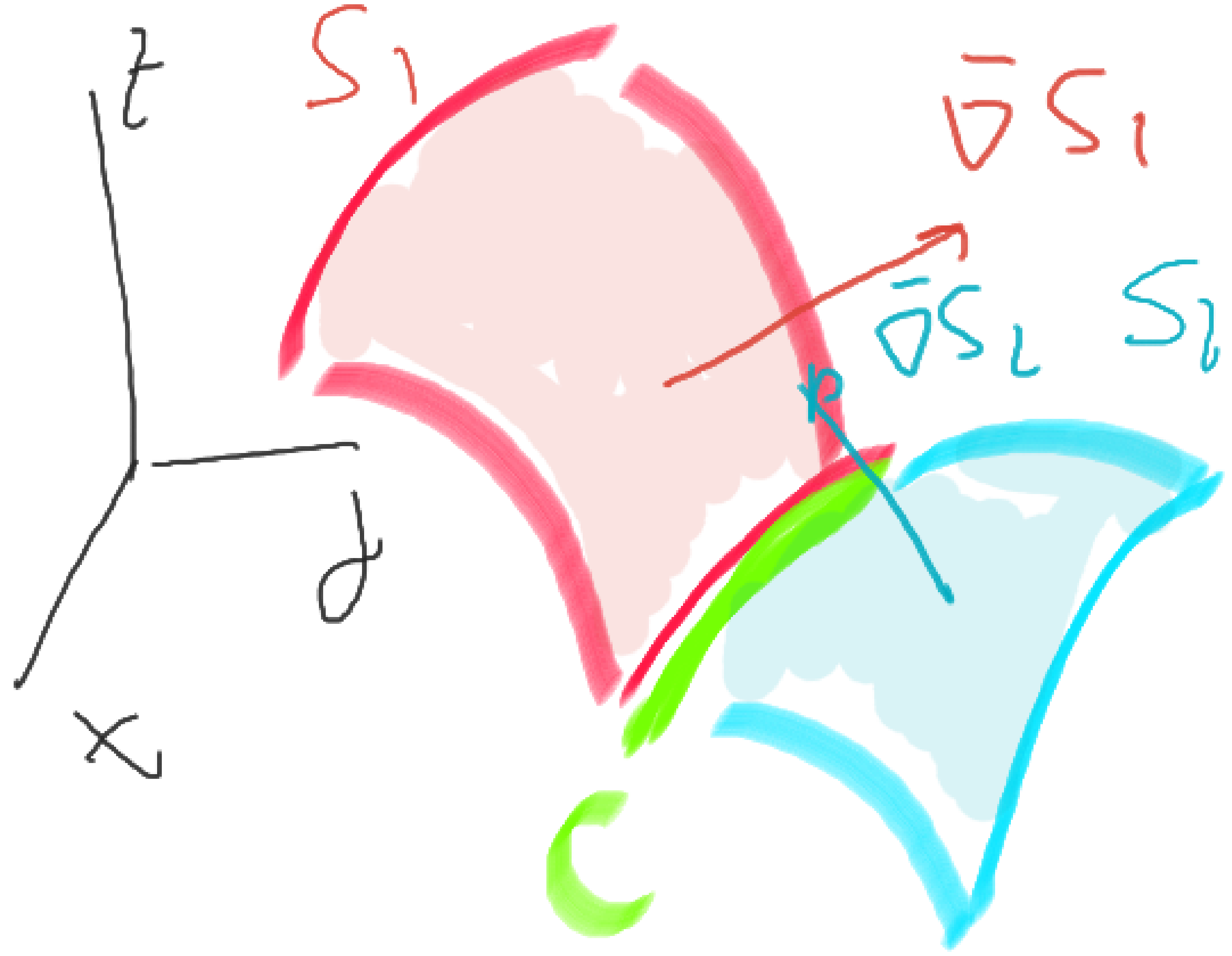
P4

Sea C la curva definida por la intersección de las superficies S_1 y S_2 de ecuaciones:

2 ✓ ✓ 1 $S_1 : x^2 + y - z = 0$ y $S_2 : x\,y\,z = 2$.

Entonces, la recta tangente a C en $(1, 1, z_0)$ interseca al plano xy en el punto: $z_0 = 2$

$(1,1,2)$ 3



$r_{Tj} : \lambda \bar{V} + \bar{P} \longrightarrow \bar{P} : (1, 1, z_0)$

$\hookrightarrow \bar{V} = \bar{\nabla} S_1 \times \bar{\nabla} S_2$

$\bar{\nabla} S_1 = (2x, 1, -1) \Rightarrow \bar{\nabla} S_1 = (2, 1, -1)$

$\bar{\nabla} S_2 = (yz, xz, xy) \Rightarrow \bar{\nabla} S_2 = (2, 2, 1)$

$= \bar{OS}_1 = (2,1,-1)$

$\Rightarrow \bar{OS}_2 = (2,2,1)$

Input interpretation

$(2, 1, -1) \times (2, 2, 1)$

Result

$(3, -4, 2)$

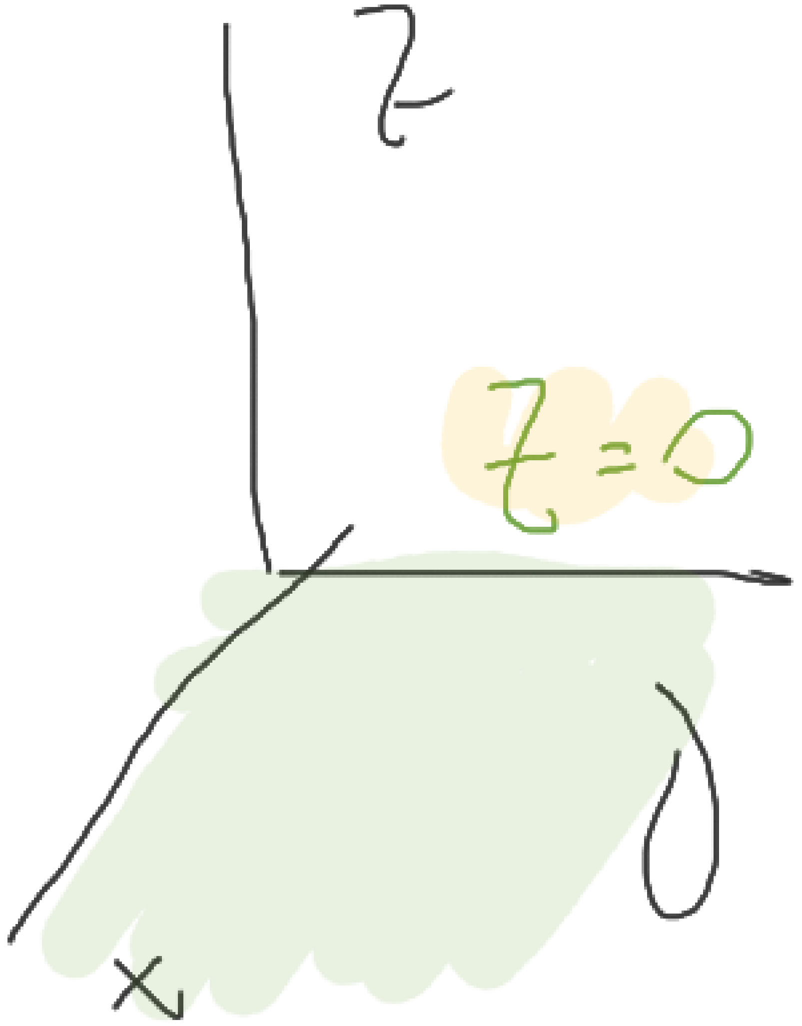
$$\bar{V} = \bar{OS}_1 \times \bar{OS}_2 \qquad \bar{V} = (2,1,-1) \times (2,2,1)$$

$$\Gamma_{xy} : \bar{\lambda}\,\bar{V} + \bar{P}$$

interseca al plano xy

$$\bar{\Gamma}_{xy} : t(3,-4,2) + (1,1,2)$$

$$(\underbrace{3t+1}_{x},\ \underbrace{-4t+1}_{y},\ \underbrace{2t+2}_{z})$$



$$2t+2=0$$

$$2t=-2$$

$$t=-1$$

$$\left(\underbrace{3t+1}_{x},\ \underbrace{-4t+1}_{y},\ \underbrace{2t+2}_{z}\right)=(-2,5,0)$$

e. $\vec{P}=(-2,5,0)$

$\vec{X} = (u^2 + v\,, u^2 v\,, u^2 - v)$ con $(u, v) \in (-1, 3) \times (2, 4)$

Contiene al punto $(2, 2, 2)$

$u^2 + v = 2$

$u^2 v = 2$

$u^2 - v = 2$

Solutions

(no solutions exist)

$z = 0$

c. Tiene un único punto donde el plano tangente es paralelo al xy

$$\left.\begin{array}{l} \Pi_1 \parallel \Pi_2 \Rightarrow N_1 = N_2 \\ \Pi_{Tg} = \bar{N} \cdot (\bar{x} - \bar{x}_0) = 0 ; \end{array}\right\} \left|\begin{array}{l} \bar{N} = (0, 0, 1) \\ \bar{N} = \bar{X}_u \times \bar{X}_v \end{array}\right|$$

$$\vec{X} = (u^2 + v,\ u^2 v,\ u^2 - v)$$

$$\bar{X}_u = (2u, 2uv, 2u)$$

$$\bar{X}_v = (1, u^2, -1)$$

$$\bar{N} = \bar{X}_u \times \bar{X}_v = \left((-2u^3 - 2uv, 4u, 2u^3 - 2uv) \right)$$

$$\bar{N} = (0, 0, 1)$$

$$\begin{cases} -2u^3 - 2uv = 0 \\ 4u = 0 \end{cases}$$

$$-\cancel{2u}(u^2 + v) = 0$$

$$u = 0 \qquad v = 0$$

$$2u^3 - 2uv = 0$$

Input interpretation

$$(2u, 2uv, 2u) \times (1, u^2, -1)$$

Result

$$(-2u^3 - 2uv, 4u, 2u^3 - 2uv)$$

e. Es un trozo de la superficie con ecuación cartesiana $y = x^2 - z^2$

$$\vec{X} = (\underbrace{u^2 + v}_{x}, \underbrace{u^2 v}_{y}, \underbrace{u^2 - v}_{z})$$

$$u^2 v = (u^2 + v)^2 - (u^2 - v)^2$$

$$u^2 v = 4u^2 v$$

$$x = u^2 + v$$

$$z = u^2 - v$$

$$x + z = 2u^2$$

$$y = u^2 v$$

$$y = \underbrace{(x + z)}_{2} \cdot v$$

d. Con esta parametrización, todos sus puntos son regulares