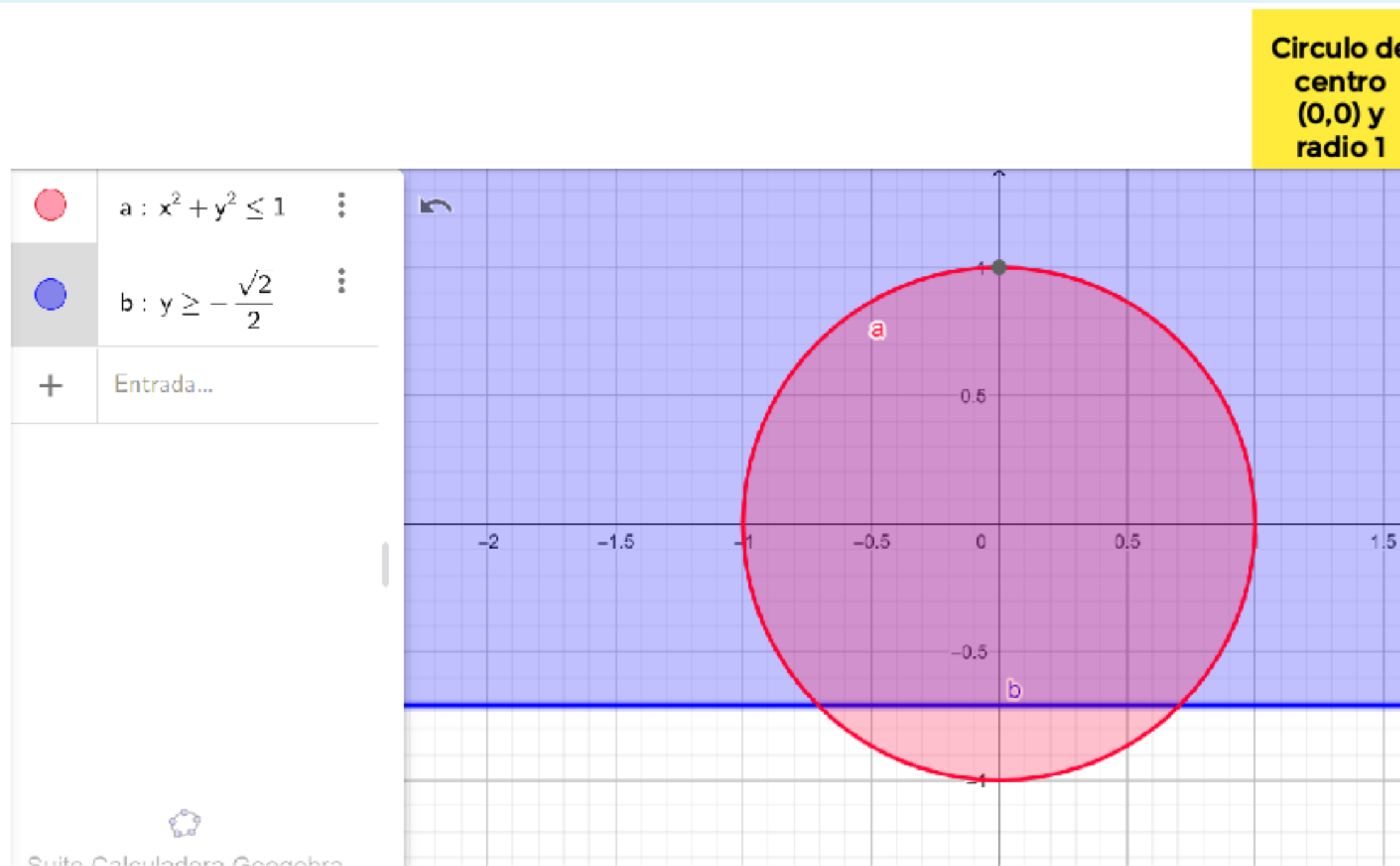


Dada $f(x, y) = 2xy + 1$ definida en $A = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 1 \wedge y \geq -\sqrt{2}/2\}$, se puede afirmar que.



Circulo de centro (0,0) y radio 1

recta con y constante

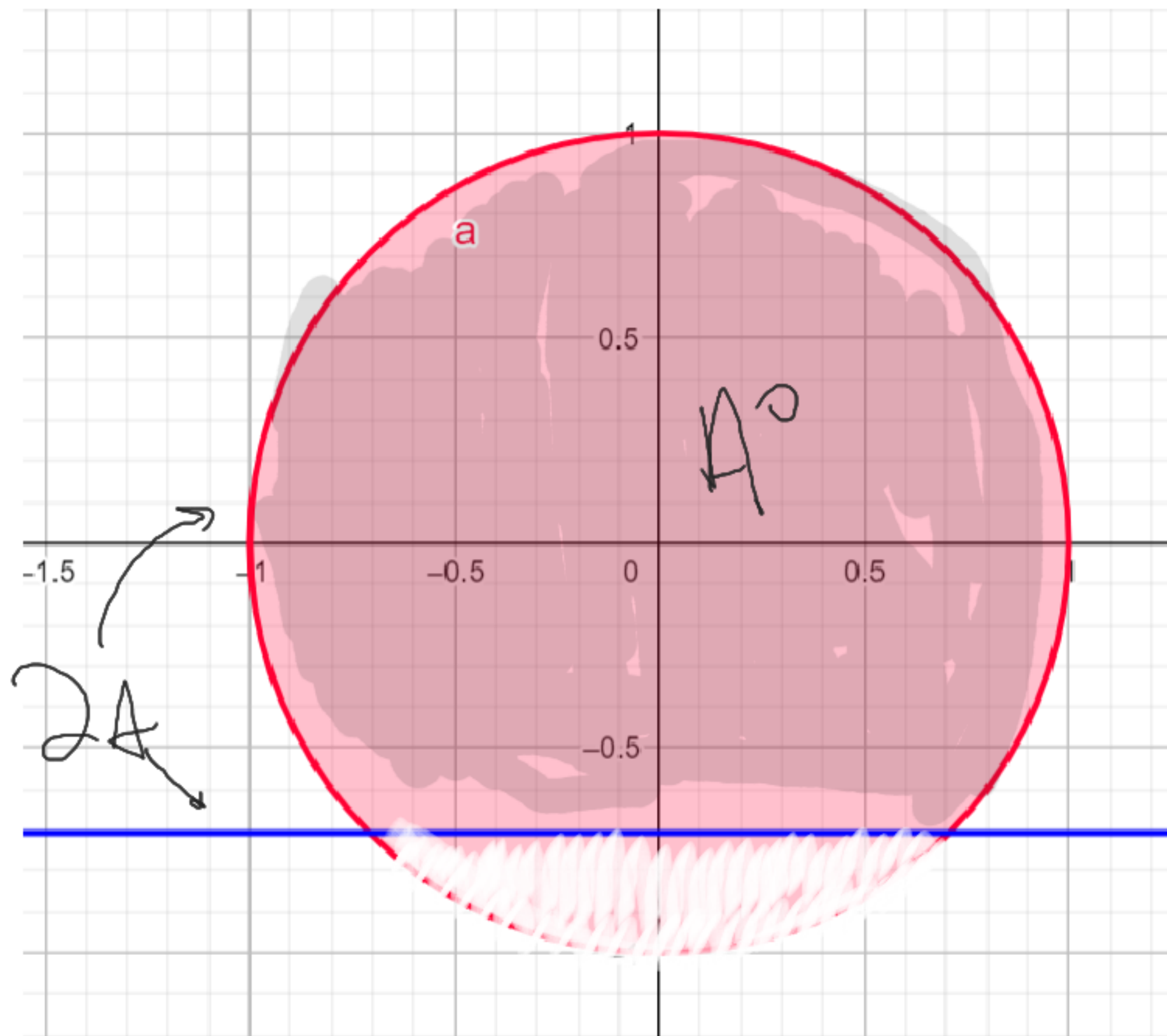
Segmento 1

$2A : \left\{ \begin{array}{l} \dots \\ \dots \end{array} \right.$

Handwritten notes in pink: $x \in [-1, 1]$ and $y \in \left[-\frac{\sqrt{2}}{2}, 1\right]$

Handwritten notes in light blue: $y = -\frac{\sqrt{2}}{2}$ and $x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

Segmento 2



$\underbrace{C \cap \partial A}$

$$f(x, y) = 2xy + 1$$

$$y = -\sqrt{2} \\ x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$$

$$\left| \left(y = -\sqrt{2} \right) \right| = 2x \left(-\frac{\sqrt{2}}{2} \right) + 1$$

$$\left| (x) \right| = -\sqrt{2}x + 1 \\ x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$$

$$f(x) = -\sqrt{2}x + 1$$

$$x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right]$$

$$f(x) = -\sqrt{2} \rightarrow \text{no se cumple}$$

$$x = -\frac{\sqrt{2}}{2}$$

$$f\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) + 1$$

$$f\left(-\frac{\sqrt{2}}{2}\right) = 2$$

$$f\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) + 1 = 0$$

$$x = \frac{\sqrt{2}}{2}$$

$C \cap \partial A$

$$f(x, y) = 2xy + 1$$

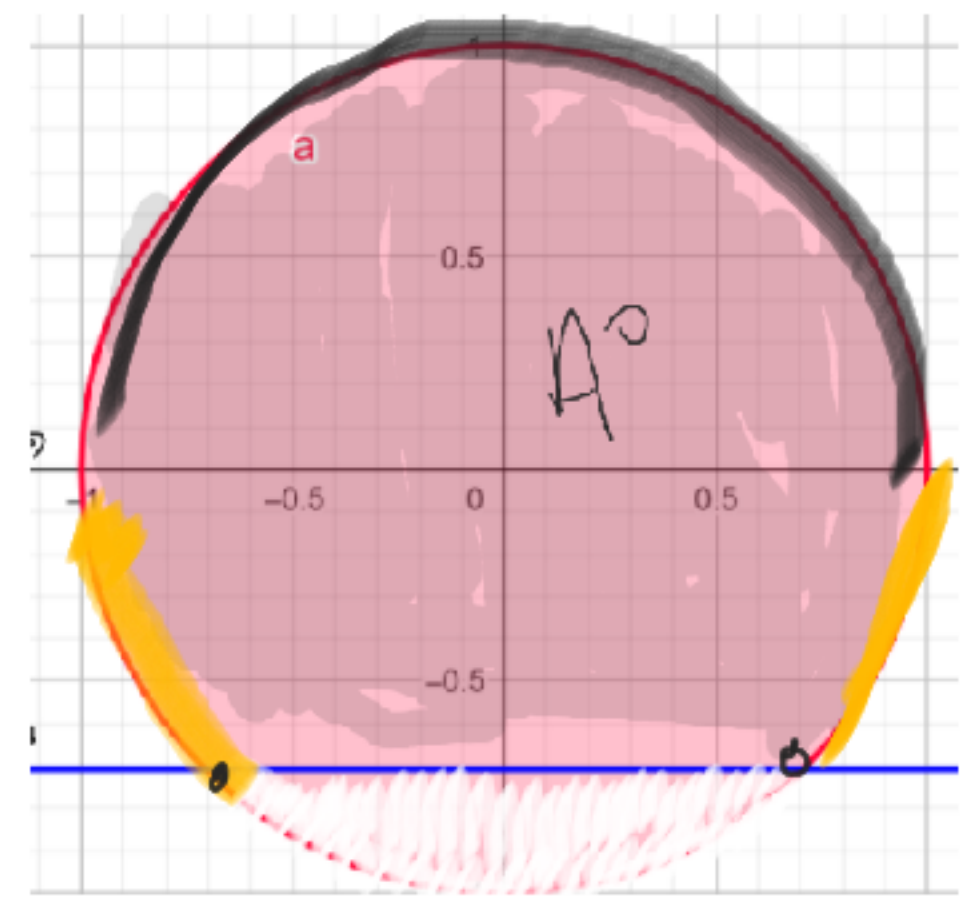
$x^2 + y^2 = 1$

$y \in \left[-\sqrt{1-x^2}, \sqrt{1-x^2} \right]$

$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y_2 = -\sqrt{1-x^2}$$
$$y_1 = \sqrt{1-x^2}$$

$y \in \left[-\sqrt{1-x^2}, \sqrt{1-x^2} \right]$

$y \in [0, 1]$



$$f(x, y) = 2xy + 1$$

$$g_2 = -\sqrt{1-x^2}$$

$$g_2 \left[-\frac{\sqrt{2}}{2}, 0 \right]$$

$$-g_2' = 0$$

$$\sqrt{1-x^2} - \frac{x^2}{2\sqrt{1-x^2}} = 0$$

$$g_2(x) = -2x\sqrt{1-x^2} + 1$$

$$g_2'(x) = -2 \left(\sqrt{1-x^2} + \frac{x \cdot (-2x)}{2\sqrt{1-x^2}} \right)$$

$$\frac{\cancel{x^2}}{\sqrt{1-x^2}} = \sqrt{1-x^2}$$

$$x^2 = (\sqrt{1-x^2})^2$$

$$x^2 = 1-x^2$$

$$2x^2 = 1$$

$$x^2 = 1/2$$

$$x = \pm \sqrt{1/2}$$

$$x_1 = \frac{1}{\sqrt{2}}, \quad y = -\frac{1}{\sqrt{2}} \rightarrow \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$$

$$x_2 = -\frac{1}{\sqrt{2}}, \quad y = \frac{1}{\sqrt{2}} \rightarrow \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$x_3 = 1, \quad y = 0 \rightarrow (1, 0)$$

$$x_4 = 1, \quad y = 0 \rightarrow (1, 0)$$

$$y_1 = \sqrt{1-x^2}$$

$$y \in [0, 1]$$

$$-2(\) = 0$$

$$\sqrt{1-x^2} - x^2 = 0$$
$$2\sqrt{1-x^2}$$

$$f(x, y) = 2xy + 1$$

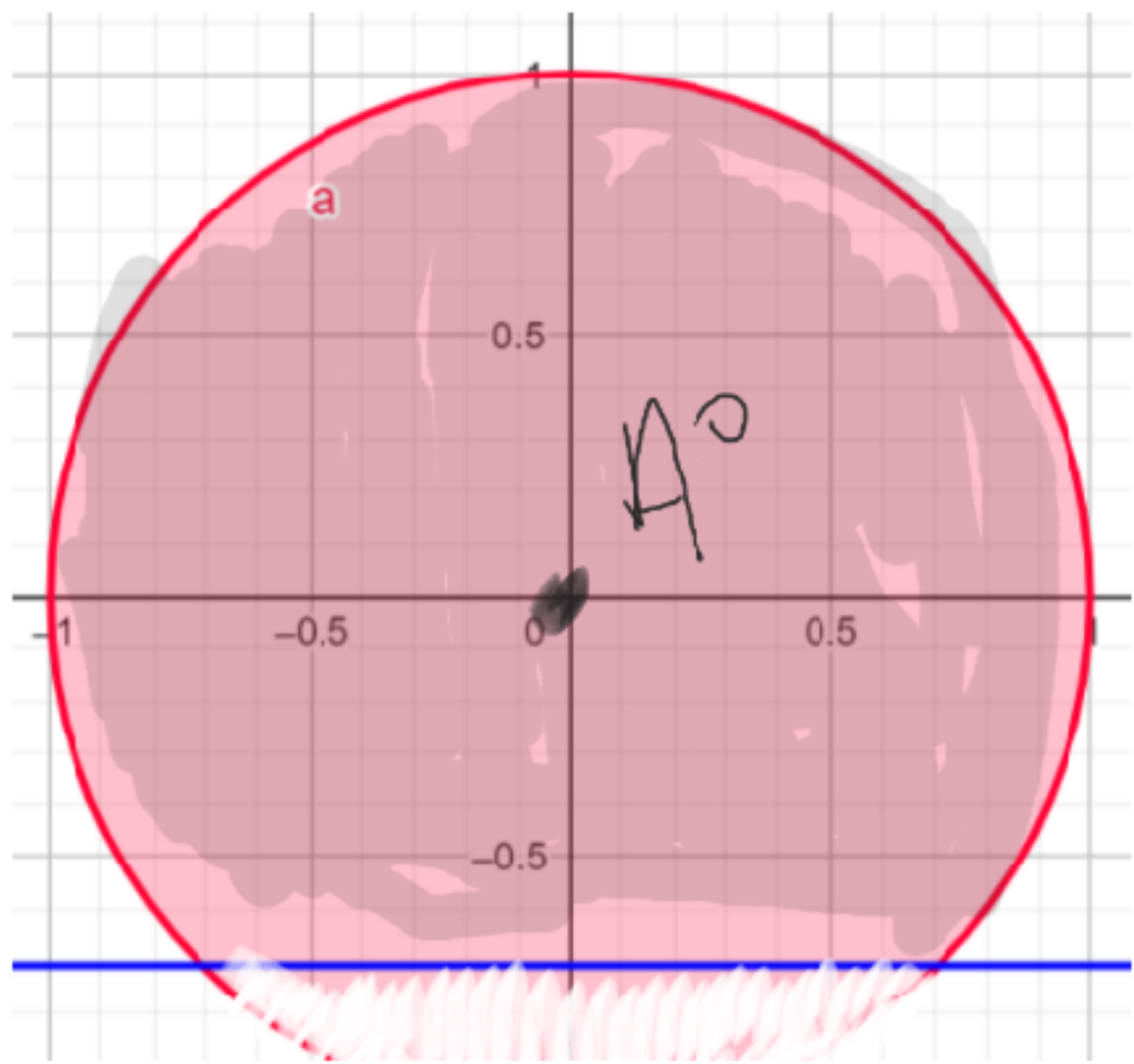
$$f(x) = -2x\sqrt{1-x^2} + 1$$

$$f'(x) = -2 \left(\sqrt{1-x^2} + \frac{x \cdot (-2x)}{2\sqrt{1-x^2}} \right)$$

$$\frac{x^2}{\sqrt{1-x^2}} = \sqrt{1-x^2}$$
$$x^2 = (\sqrt{1-x^2})^2$$

$$x^2 = 1-x^2$$
$$2x^2 = 1 \quad x = \frac{1}{\sqrt{2}}, y = -\frac{1}{\sqrt{2}} \rightarrow (1/\sqrt{2}, -1/\sqrt{2})$$

$$x^2 = 1/4 \quad x = -\frac{1}{2}, y = \frac{1}{2} \rightarrow (-1/2, 1/2)$$



$$f(x, y) = 2xy + 1$$

$$x = 2 = 0$$

$$y = 2x = 0$$

$$y = 0$$

$$x = 0$$

$$P_S = (2, 0)$$

$$\in A^0!$$

c

- $(1/\sqrt{2}, -1/\sqrt{2})$
- $(-1/\sqrt{2}, -1/\sqrt{2})$
- $(-1, 0)$
- $(1, 0)$
- $(2, 0)$**

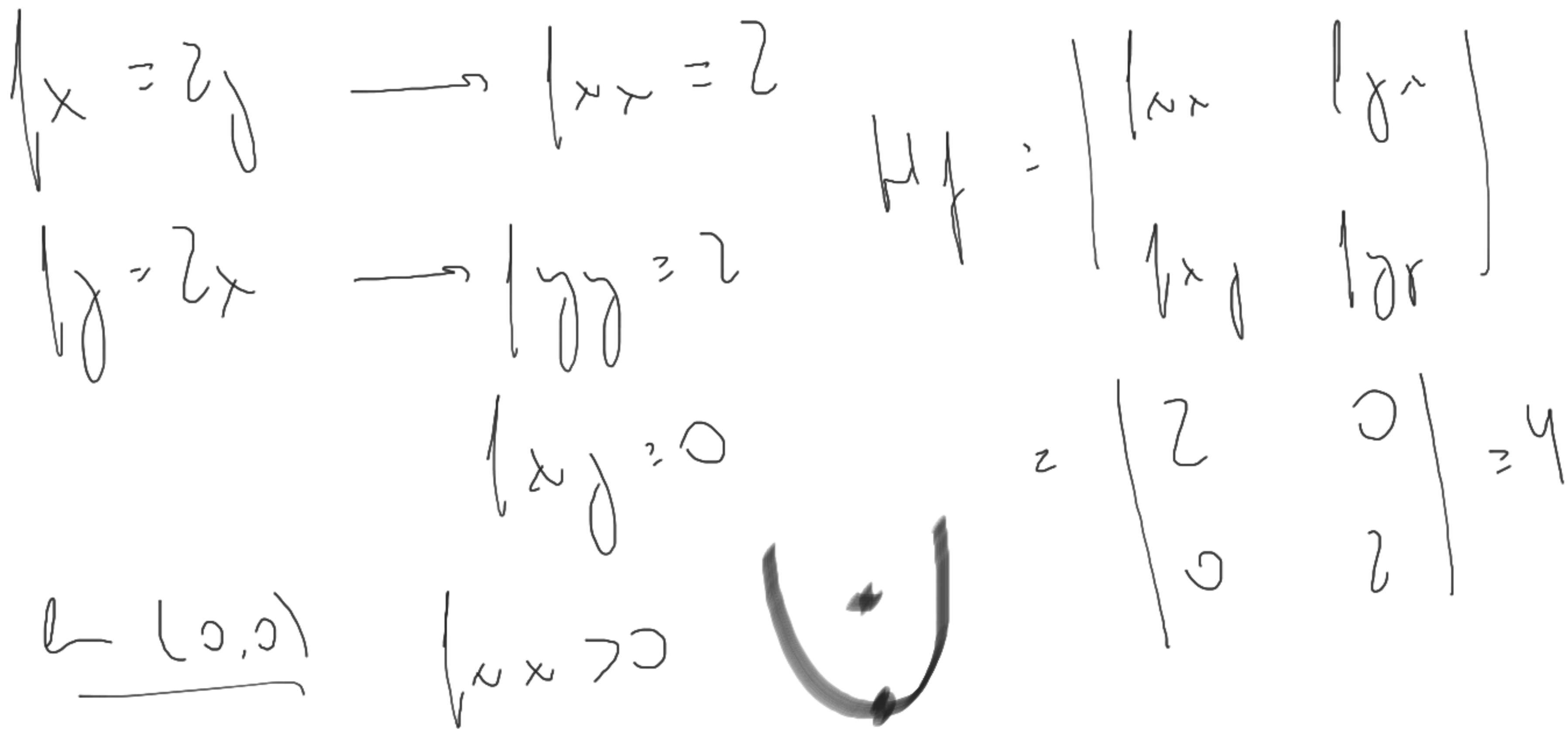
0 **max**

2 **max**

1

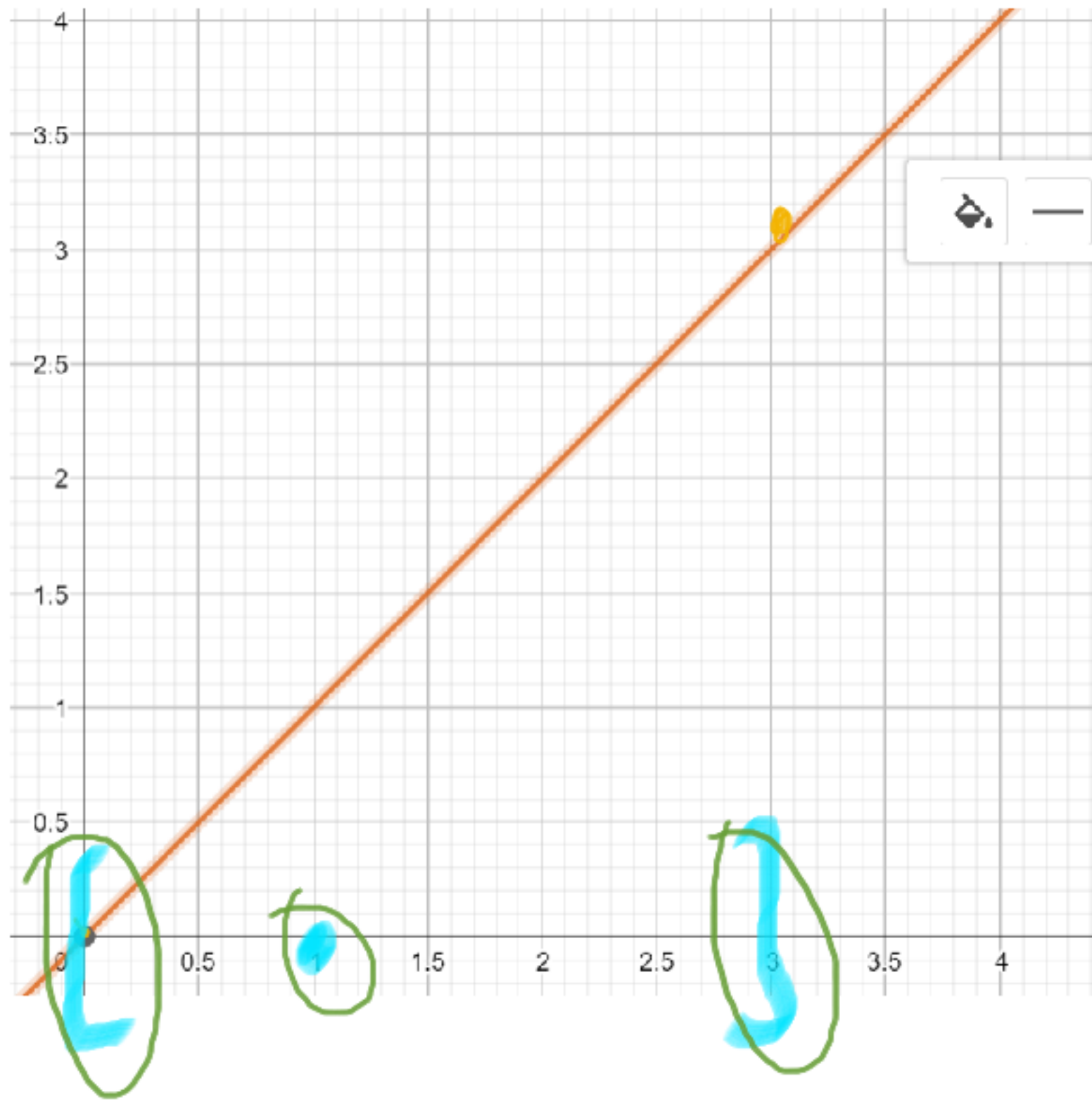
1

1



d. f produce extremos absolutos en A y $f(0,0)$ no es extremo local

Considere los valores de $f(x, y) = (x - 1)^2 + (y - 1)^2$ evaluada en puntos del segmento que une al origen de coordenadas con el punto $(3, 3)$, los correspondientes extremos absolutos se producen en los puntos:



$$y = x, \quad x \in [0, 3]$$

$$f(x) = (x - 1)^2 + (x - 1)^2$$

$$f(x) = 2(x - 1)^2$$

$$f'(x) = 2 \cdot 2(x - 1)^1 \cdot 1$$

$$f'(x) = 2 \cdot 2(x-1)^1 \cdot 1 = 0$$

$$y = x$$

$(0, 0)$	0
----------	---

$$(x-1) \cdot 4 = 0$$

$(1, 1)$	1
----------	---

$$x-1 = 0$$

$(3, 3)$	3
----------	---

$$\boxed{x = 1}$$

$$f(x) = 2(x-1)^2$$

2	
---	--

0 min

8 max

c. (1, 1) y (3, 3)

Sean las superficies $S_1 = \{(x, y, z) \in \mathbb{R}^3 / z = 4 - x^2\}$ y $S_2 = \{(x, y, z) \in \mathbb{R}^3 / z = 4 - y^2\}$ y sea la curva $C = S_1 \cap S_2$ que contiene al punto $P_0 = (1, 1, 3)$. Entonces, se puede afirmar que.

$$S_1: z = 4 - x^2$$

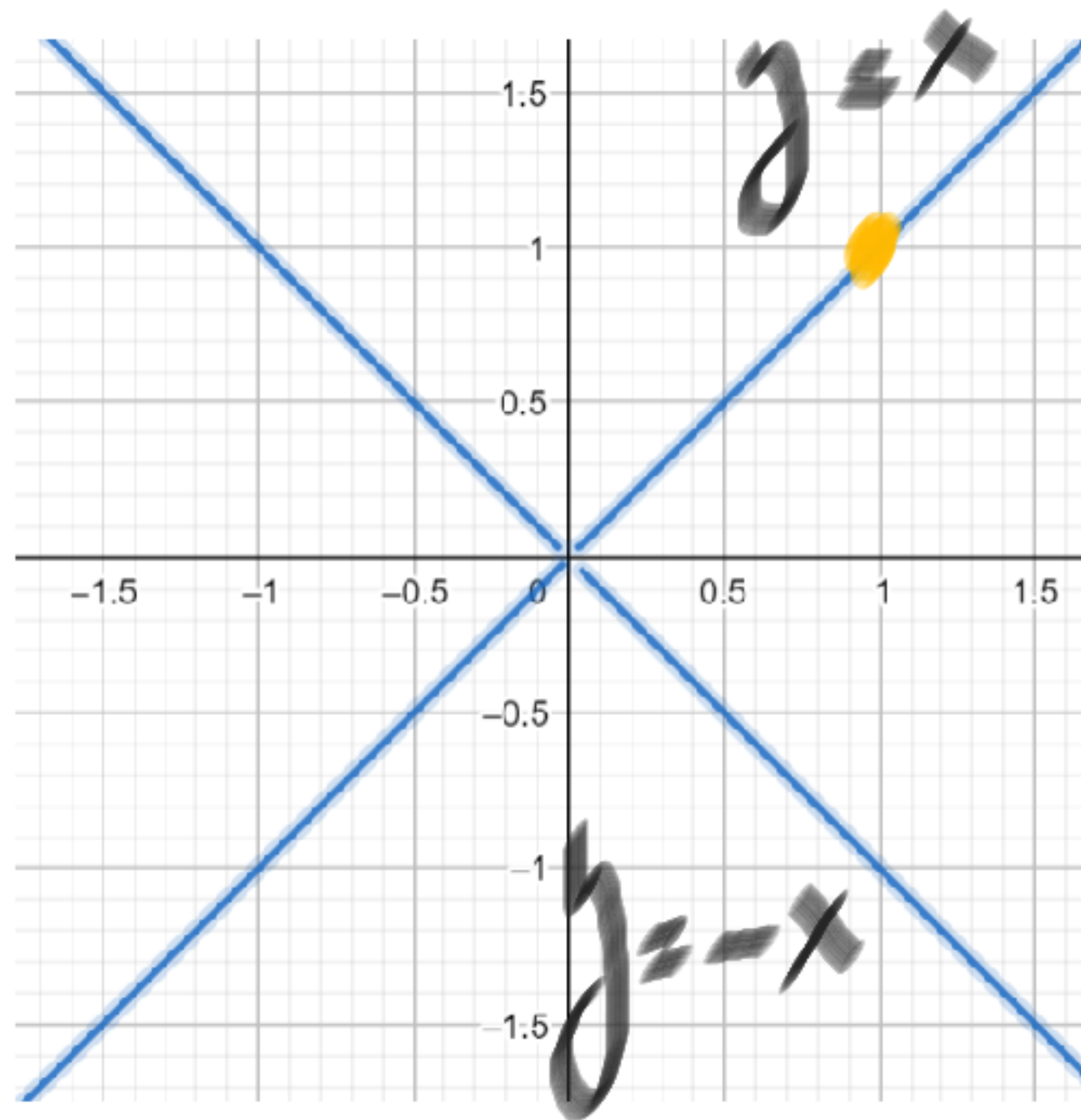
$$S_2: z = 4 - y^2$$

$$z = z$$

$$4 - x^2 = 4 - y^2$$

$$x^2 = y^2$$

$$|x| = |y|$$



$$C: \begin{cases} y = x \\ z = 4 - x^2 \end{cases}$$

$$C(x) = (x, y, z)$$

$$\bar{C}(x) = (x, x, 4 - x^2)$$

$$x = 1$$

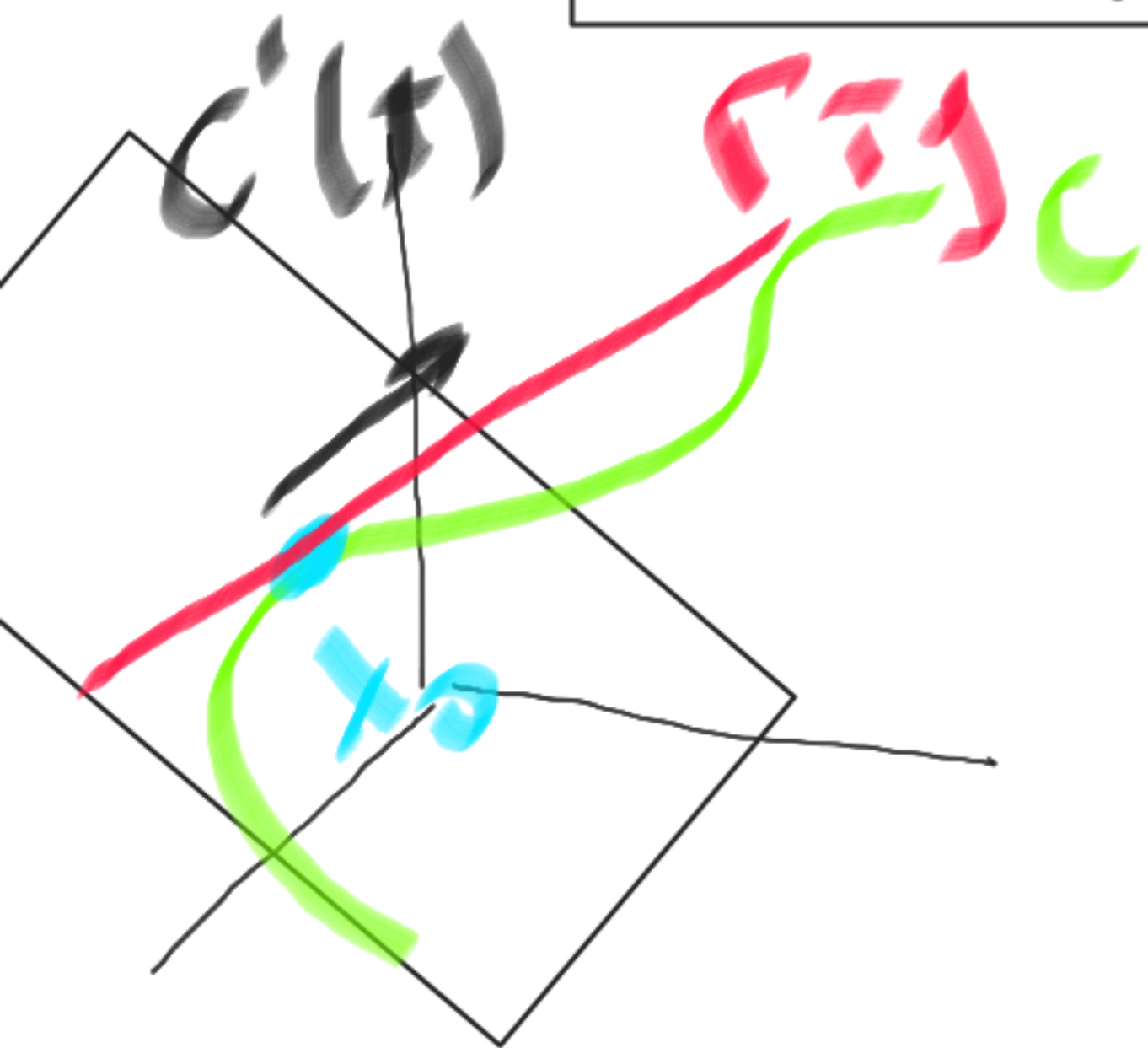
$$\bar{C}(\lambda) = (\lambda, \lambda, 4 - \lambda^2)$$

$$x = 1$$

$$\bar{C}'(\lambda) = (1, 1, -2\lambda)$$

$$\left[\begin{array}{c} \bar{y} \\ y \end{array} \right]_C : \pi \cdot \bar{C}'(1) + C(1)$$

$$x_0$$



$$\left[\begin{array}{c} \bar{y} \\ y \end{array} \right]_C : \pi (1, 1, -2) + (1, 1, 3)$$

$$\bar{N} \cdot (\bar{x} - \bar{x}_0) = 0$$

$$(1, 1, -2) \cdot ((x, y, z) - (1, 1, 3)) = 0$$

La recta tangente a C en P_0 interseca al plano xy en el punto $(3, 3, 0)$

$$(1, 1, -2) \cdot ((x, y, z) - (1, 1, 3)) = 0$$

$$x + y - 2z - 1 - 1 + 6 = 0$$

$$\Pi: x + y - 2z = -4$$

$$r_{xy}^C: x=1: \tau(1, 1, -2) + (1, 1, 3)$$

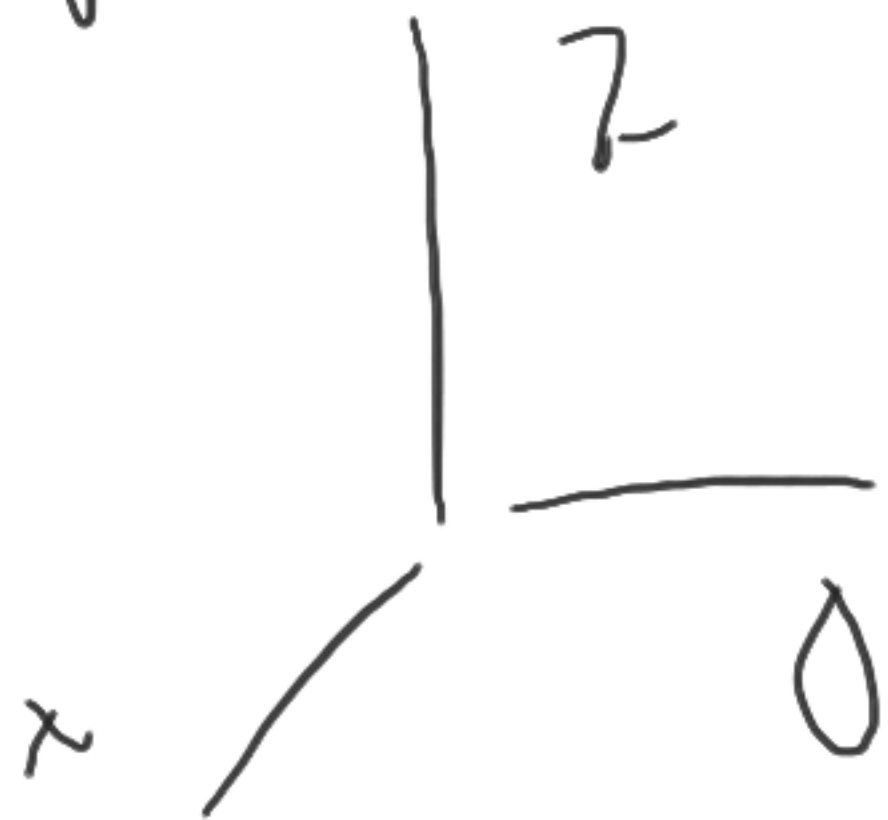
$$z=0 \quad (3, 3, 0)$$
$$\vec{r}(\tau) = (\tau+1, \tau+1, -2\tau+3)$$

$$\tau = 2$$

c. El plano normal a C en P_0 no tiene punto en común con el eje z

$$\Pi: x + y - 2z = -4$$

by $z \rightarrow x=0, y=0$



$$0 + 0 - 2z = -4$$

$$\boxed{z = 2}$$

d. La recta tangente a C en P_0 interseca al eje z en el punto $(0, 0, 5)$

$$\vec{r}(t) = (t+1, t+1, -2t+3)$$

$$t = -1$$

$$-2(-1) + 3 = 5$$



Dada $f(x, y) = \begin{cases} \frac{xy}{3x^2 + 2y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{xy}{3x^2 + 2y^2}$$

Result

(limit does not exist, is path dependent, or cannot be determined)

$y = mx$

$$\frac{x \cdot mx}{3x^2 + 2m^2x^2}$$

$$(x, mx) \rightarrow (0, 0) \quad \frac{x^2}{3x^2 + 2m^2x^2}$$

$\frac{x^2}{x^2} = \frac{m}{3 + 2m^2}$

Derivative

$$\frac{\partial}{\partial x} \left(\frac{xy}{3x^2 + 2y^2} \right) = \frac{y(2y^2 - 3x^2)}{(3x^2 + 2y^2)^2}$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{y(2y^2 - 3x^2)}{(3x^2 + 2y^2)^2}$$

Result

(limit does not exist)

(value may depend on x, y path in complex space)

(6) **Definición.** Sean $f: A \subset \mathbb{R}^2 \rightarrow \mathbb{R}$, A abierto y $\mathbf{x}_0 \in A$. Decimos que f es *diferenciable* en \mathbf{x}_0 si existen números a y b tales que

$$\lim_{\mathbf{x} \rightarrow \mathbf{x}_0} \frac{f(\mathbf{x}) - f(\mathbf{x}_0) - a(x - x_0) - b(y - y_0)}{\|\mathbf{x} - \mathbf{x}_0\|} = 0.$$

(7) **Proposición.** Si $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ es diferenciable en \mathbf{x}_0 , entonces existen las derivadas parciales de f en \mathbf{x}_0 y $a = f_x(\mathbf{x}_0)$, $b = f_y(\mathbf{x}_0)$...

Dada $f(x, y) = \begin{cases} \frac{xy}{3x^2 + 2y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$

$x = h$
 $h \rightarrow 0$
 $\frac{f(0+h, 0) - f(0, 0)}{h}$

$h \rightarrow 0$
 $\frac{0}{3h^2} = \frac{0}{h} = 0 = 0 = 0$

$y = h$
 $h \rightarrow 0$
 $\frac{f(0, 0+h) - f(0, 0)}{h} = \frac{0}{2h^2} = \frac{0}{h} = 0 = 0$

$$\lim_{\mathbf{x} \rightarrow \mathbf{x}_0} \frac{f(\mathbf{x}) - f(\mathbf{x}_0) - a(x - x_0) - b(y - y_0)}{\|\mathbf{x} - \mathbf{x}_0\|} = 0.$$

↳
 $(x, y) \rightarrow (0, 0)$

$$\frac{xy}{3x^2 + 2y^2} : \sqrt{x^2 + y^2}$$

↳
 $(x, y) \rightarrow (0, 0)$

$$\frac{xy}{(3x^2 + 2y^2) \sqrt{x^2 + y^2}}$$

$$a = \begin{cases} x \\ y \end{cases}$$

$$b = \begin{cases} x \\ y \end{cases}$$

$$\sqrt{(0, 0)} = 0$$

$$\|\mathbf{x} - \mathbf{x}_0\| =$$

$$\|\mathbf{x} + (0, 0)\| =$$

$$\|\mathbf{x}\| = \sqrt{x^2 + y^2}$$

Input

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2} (3x^2 + 2y^2)}$$

Result

(limit does not exist)

(value may depend on x, y path in complex space)

f no es
diferenciable

a. f no es continua y no es clase C^1 en el origen

La curva dada como intersección de las superficies S_1 y S_2 de ecuaciones:

$$S_1 : z = \sqrt{4 - x^2 - y^2} \quad \text{y} \quad S_2 : x^2 + y^2 = 3 \quad \text{con} \quad x \geq 0$$

se puede parametrizar mediante la función \vec{g} tal que:

$$x^2 + y^2 = 3$$

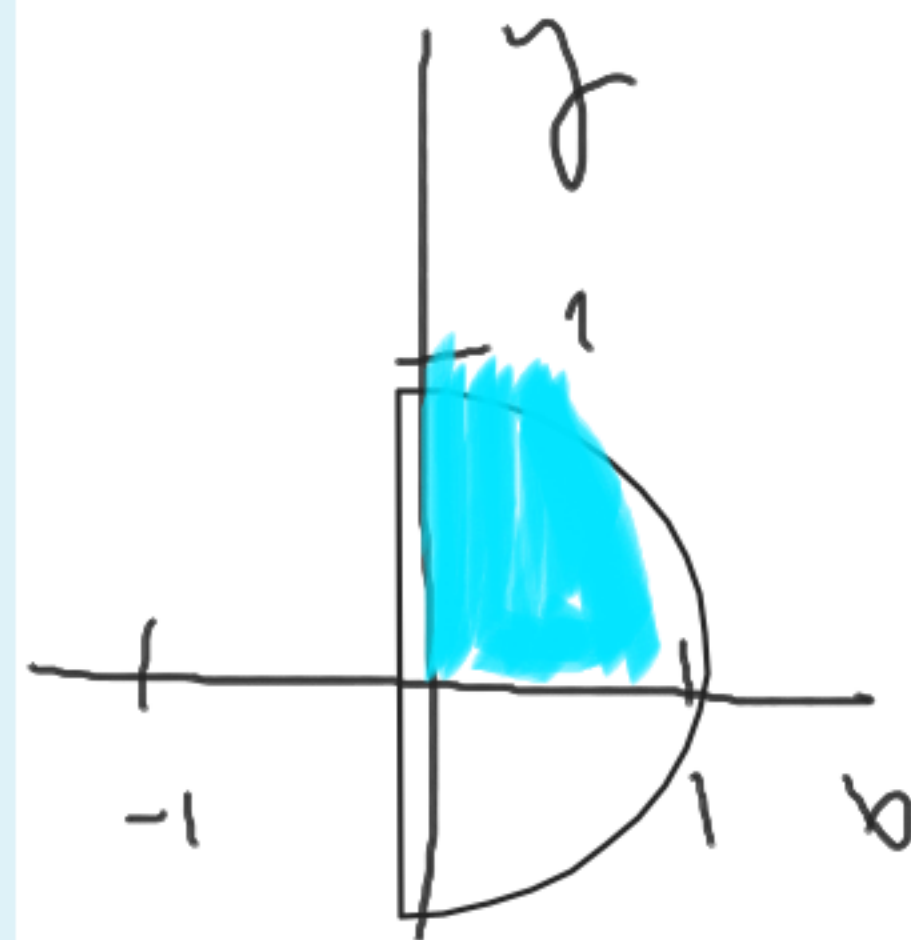
$$z = \sqrt{4 - x^2 - y^2}$$

$$z = \sqrt{4 - (x^2 + y^2)}$$

$$z = \sqrt{4 - 3}$$

$$z = 1$$

- a. $\vec{g}(x) = (x, \sqrt{3-x^2}, 1)$ con $-\sqrt{3} \leq x \leq \sqrt{3} \rightarrow$ No
- b. $\vec{g}(x) = (x, \sqrt{3-x^2}, -1)$ con $0 \leq x \leq \sqrt{3}$
- c. $\vec{g}(x) = (x, -\sqrt{3-x^2}, 1)$ con $0 \leq x \leq \sqrt{3}$
- d. Ninguna de las otras es correcta No
- e. $\vec{g}(t) = (\sqrt{3} \cos(t), \sqrt{3} \operatorname{sen}(t), 1)$ con $0 \leq t \leq \pi/2$ ●



$$x^2 + y^2 = 3$$

$$y^2 = 3 - x^2$$

$$y = \sqrt{3 - x^2}$$

$$y = -\sqrt{3 - x^2}$$