Sea $y \in C^{\infty}(\mathbb{R})$ tal que $y'' + 5y' - 14y = 0$

Polinomio Característico

EDO 2°grado, homogénea --> polinomio caracteristico

$y \to 1$, $y' \to p$; $y'' \to p^2$; $p^2 + 5p - 14 = 0$

Solutions

$p = -7 \rightsquigarrow r_1$

$p = 2 \rightsquigarrow r_2$

$y(x) = C_1 \cdot e^{r_1 x} + C_2 \cdot e^{r_2 x}$

$$y''(x) + 5\,y'(x) - 14\,y(x) = 0$$

$$y(x) = c_1\,e^{-7x} + c_2\,e^{2x}$$

$$\lim_{x \to +\infty} y(x) = 0$$

$e^{-\infty} = 0$ $\qquad$ ∞

$$\lim_{x \to \infty} \quad y(x) = c_1 e^{-7x} + c_2 e^{2x}$$

$$y(0) = c_1 + c_2 \qquad 0$$

$$y'(x) = -7c_1 e^{-7x} + 2c_2 e^{2x}$$

$$y'(0) = -7c_1 + 2c_2 \qquad 0$$

Si $\lim_{x \to \infty} y(x)$ $\quad [y(0) \;\; y'(0)]^T \in \text{gen}\{[1 \;\; -7]^T\}$

$$c_1 e^{-7x} + c_2 e^{2x} \qquad \infty \quad 0$$

$$c_2 \to 0$$

$$[y(0) \;\; y'(0)]^T \in \text{gen}\{[1 \;\; -7]^T\}$$

Valores inciales

$$[y(0) \;\; y'(0)]^T$$

$$= \{(c_1, -7c_1)\} = c_1\{(1, -7)\}$$

Sean $A \in \mathbb{R}^{3\times3}$ y $B \in \mathbb{R}^{3\times4}$ dos matrices tales que $AB = \begin{bmatrix} -4 & 4 & 4 & 2 \\ -7 & 7 & 7 & 1 \\ 8 & -8 & -8 & 9 \end{bmatrix}$,

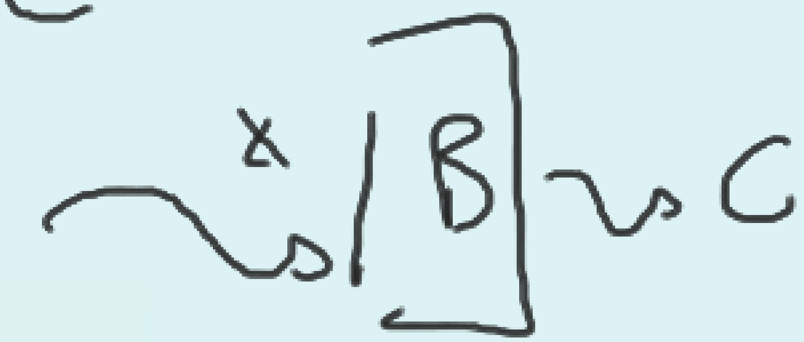
donde $\text{rango}(A) = 3$ y B satisface que

$B\begin{bmatrix}-1 & 0 & -1 & 1\end{bmatrix}^T = \begin{bmatrix}2 & 1 & 9\end{bmatrix}^T$,

$B\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^T = \begin{bmatrix}6 & 8 & 1\end{bmatrix}^T$.

El conjunto solución de la ecuación $Bx = \begin{bmatrix}-4 & -7 & 8\end{bmatrix}^T$ es ...

$Bx = C$



$(-4,-7,8) = a\cdot(2,1,9) + b\cdot(6,8,1)$

$\bar{v} \in Nu(B)$

$x = a\cdot(-1,0,-1,0) + b\cdot(1,1,1,1) + \bar{v}$,

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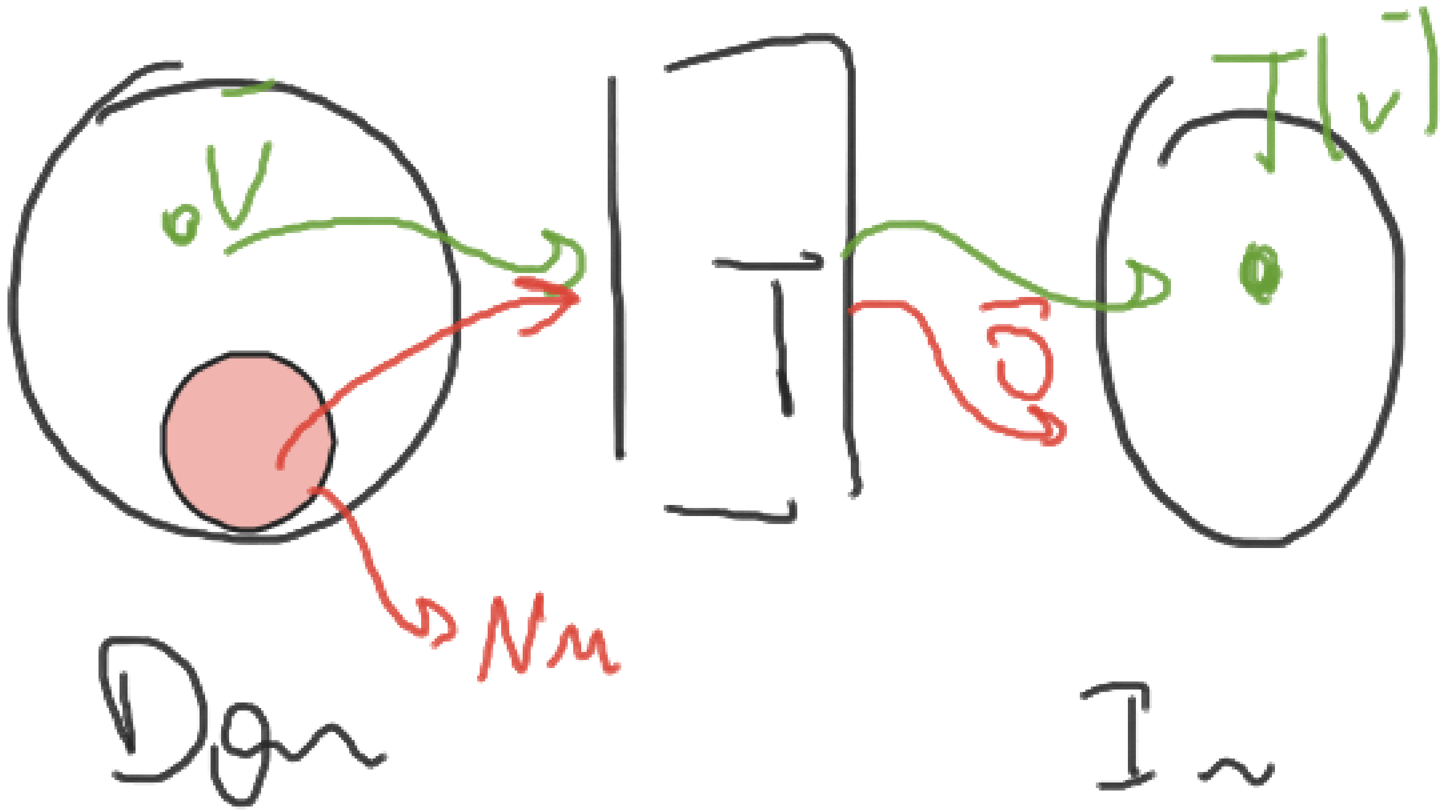
$$(-4,-7,8) = a\cdot(2,1,9) + b\cdot(6,8,1) \qquad a = 1, \quad b = -1$$

$$\bar{v} \in Nu(B)$$

$$\left. \begin{aligned} B\bar{v} &= \bar{0} \\ \underbrace{A}_{3\times 3}\,\underbrace{B\bar{v}}_{3\times 1} &= A\cdot\underbrace{\bar{0}}_{3\times 1} \end{aligned} \right\} \quad Nu(B) = Nu(4)$$

$$AB = \begin{bmatrix} -4 & 4 & 4 & 2 \\ -7 & 7 & 7 & 1 \\ 8 & -8 & -8 & 9 \end{bmatrix}$$



WolframAlpha

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{{-4,4,4,2},{-7,7,7,1},{8,-8,-8,9}}*{{x_1},{x_2},{x_3},{x_4}} = {{0},{0},{0}}
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$$Nu: \quad x_3 = x_1 - x_2, \quad x_4 = 0$$

N_u $x_3 = x_1 - x_2\,, \quad x_4 = 0$ $\quad (x_1, x_2, x_1 - x_2, 0)$

$\checkmark\ N_u = \{(0, 1, -1, 0)\ (1, 0, 1, 0)\}$ $\quad \bar{v} = \alpha(1,0,1,0) + \beta(1,1,0,0)$

$(-4, -7, 8) = a\cdot(2,1,9) + b\cdot(6,8,1)$ $\quad a = 1, \quad b = -1$

$x = a\cdot(-1, 0, -1, 0) + b\cdot(1,1,1,1) + \bar{v},$ $\quad (a = 1,\ b = -1)$

c. $\left\{ \begin{bmatrix} -2 \\ -1 \\ -2 \\ 0 \end{bmatrix} + a \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}$

En $\mathbb{R}^3$ con el producto interno $\langle\cdot,\cdot\rangle$ definido por

$$\langle x, y\rangle = y^T \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} x\,,$$

se considera la funcional lineal $\phi : \mathbb{R}^3 \to \mathbb{R}$ definida por

$$\phi(x) = 2x_1 + 5x_2 + 3x_3$$

El único vector $v \in \mathbb{R}^3$ tal que $\phi(x) = \langle x, v\rangle$ para todo $x \in \mathbb{R}^3$ es

a. $v = \begin{bmatrix} -3 & 5 & 1 \end{bmatrix}^T$

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1. En $\mathbb{R}^3$ con el producto interno definido por

$$\langle x, y\rangle = y^T \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} x$$

$\to A$ 1×3 3×1

se considera el funcional lineal $\phi : \mathbb{R}^3 \to \mathbb{R}$ definido por $\phi(x) = 2x_1 + 5x_2 + 3x_3$ El único vector $v \in \mathbb{R}^3 : \ \phi(x) = \langle x, v\rangle, \ \forall x \in \mathbb{R}^3$ es

$$\phi(x) = \begin{pmatrix} 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\langle x, v\rangle = \phi(x)$$

$$y^T A = (2, 5, 3)$$

$$\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} A = (2.5.3)$$

$$\langle x, v\rangle = v^T A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\underbrace{(y_1, y_2, y_3)}_{1\times 3} \underbrace{\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{3\times 3} = \underbrace{(2, 5, 3)}_{1\times 3}$$

- $3y_1 + 2y_2 + y_3 = 2$
- $2y_1 + y_2 + y_3 = 5$
- $y_1 + y_2 + y_3 = 3$

Solution

$$y_1 = -3\,, \quad y_2 = 5\,, \quad y_3 = 1$$

$$y = (-3, 5, 1)$$

a. $v = \begin{bmatrix} -3 & 5 & 1 \end{bmatrix}^T$

Sea $(V, \langle \cdot, \cdot \rangle)$ un $\mathbb{R}$ -espacio euclídeo de dimensión 3, y sea $\{v_1, v_2, v_3\}$ una base ortonormal de V.

La distancia del vector $2v_1 + 5v_2$ al subespacio gen $\{v_1 + 3v_2, 2v_1 + 3v_3\}$ es ...

$B = \{(1,3,0); (2,0,3)\}$ $\quad V = (2,5,0)$ $\quad \bar{\omega} \in B^\perp$

$B^\perp$ (a,b,c).(1,3,0)=0, (a,b,c).(2,0,3)=0 $\longrightarrow$ $B^\perp = \{(a, -\frac{a}{3}, -\frac{2}{3}a)\} = \{(3,-1,2)\}$

$$d(x, S) = |P_{S^\perp}(x)|$$

$$b = -\frac{a}{3}, \quad c = -\frac{2a}{3}$$

$$P_S(x) \doteq \sum_{i=1}^{q} \frac{< v_i, x >}{< v_i, v_i >} \cdot v_i$$

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$$P_S(x) \doteq \sum_{i=1}^{q} \frac{< v_i, x >}{< v_i, v_i >} \cdot v_i = \frac{(2,5,0)\cdot(3,-1,2)}{\|(3,-1,2)\|^2} \cdot (3,-1,2)$$

$$= \frac{6-5}{9+1+4} \cdot (3,-1,2) = \left(\frac{3}{14}, \frac{-1}{14}, \frac{2}{14}\right)$$

$$d(x,S) = |P_{S^\perp}(x)| = \left|\frac{(3,-1,2)}{14}\right| = \frac{\sqrt{14}}{14}$$

De acuerdo con la técnica de mínimos cuadrados, la recta que mejor ajusta los siguientes datos

x	-1	0	1	2	3
y	1	3	7	10	13

$$y = a_0 + a_1 x$$

A	
a_0	a_1*x
1	-1
1	0
1	1
1	2
1	3

$A^{\#}$

Exact result

$$\frac{1}{10}\begin{pmatrix} 4 & 3 & 2 & 1 & 0 \\ -2 & -1 & 0 & 1 & 2 \end{pmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}}_{\hat{x}} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{y}$$

$$\boxed{\hat{x} = A^{\#} \cdot y}$$

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$$\boxed{\hat{x} = A^{\#} \cdot y}$$

$$\frac{1}{10}\begin{pmatrix} 4 & 3 & 2 & 1 & 0 \\ -2 & -1 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 7 \\ 10 \\ 13 \end{pmatrix} = \begin{pmatrix} \frac{37}{10} \\ \frac{31}{10} \end{pmatrix} \begin{matrix} \to a_0 \\ \to a_1 \end{matrix}$$

$$y = \frac{37}{10} + \frac{31}{10}x$$

$$y = \frac{1}{10}(37 + 31x)$$

Sean $\mathbb{U}$ y $\mathbb{S}$ los subespacios de $\mathbb{R}_3[x]$ definidos por

$\mathbb{U} = \{p \in \mathbb{R}_3[x] : p'(3) = 0\}$ y $\mathbb{S} = \text{gen}\{1 - 6x + x^2, 2 - 27x + x^3\}$

Un subespacio $\mathbb{T}$ de $\mathbb{R}_3[x]$ tal que $\mathbb{S} \oplus \mathbb{T} = \mathbb{U}$ es ...

$E\{1, x, x^2, x^3\}$

U

$p(x) = u_0 + u_1 x + u_2 x^2 + u_3 x^3$

$p'(x) = u_1 + 2u_2 x + 3u_3 x^2$

$p'(3) = u_1 + 6u_2 + 27u_3 = 0$

$\boxed{u_1 = -6u_2 - 27u_3}$

$(u_0, u_1, u_2, u_3) =$

$= (u_0, -6u_2 - 27u_3, u_2, u_3)$

$U = \{(1,0,0,0), (0,-6,1,0), (0,-27,0,1)\}$

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$S = \text{gen}\{1 - 6x + x^2, 2 - 27x + x^3\}$ $S = \{(1,-6,1,0), (2,-27,0,1)\}$

$S \oplus T = U$

$U = \{(1,0,0,0), (0,-6,1,0), (0,-27,0,1)\}$

a. $T = \text{gen}\{-9x^2 + 2x^3\}$ $= (0,0,-9,2)$

b. $T = \text{gen}\{9x^2 + 2x^3\}$ $= (0,0,9,2)$

c. $T = \text{gen}\{-5 - 9x^2 + 2x^3\}$ $= (-5,0,-9,2)$

d. $T = \text{gen}\{13 + 9x^2 + 2x^3\}$ $= (13,0,9,2)$

Maximal linearly independent subset

$\{(1, 0, 0, 0), (0, -6, 1, 0), (0, -27, 0, 1)\}$

$S+T = \{(1,-6,1,0), (2,-27,0,1), (0,0,-9,2)\}$

linear independence (1,0,0,0),(0,-6,1,0),(0,-27,0,1),(1,-6,1,0),(2,-27,0,1),(0,0,-9,2)

Sea $S_a \subseteq \mathbb{R}^{2\times 2}$ el subespacio definido por

$$S_a = \text{gen}\left\{ \begin{bmatrix} -3 & a-5 \\ a-8 & -3 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} a-2 & a-7 \\ a-7 & 0 \end{bmatrix} \right\}, \text{ con } a \in \mathbb{R}$$

$$S = \{ (-3, a-5, a-8, -3) \; (1, 0, 1, 1) \; (a-2, a-7, a-7, 0) \}$$

$$\left(\begin{array}{ccc|c} -3 & a-5 & a-8 & -3 \\ 1 & 0 & 1 & 1 \\ a-2 & a-7 & a-7 & 0 \end{array} \right)$$

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$$\begin{vmatrix} -3 & -5+a & -8+a \\ 1 & 0 & 1 \\ -2+a & -7+a & -7+a \end{vmatrix} \neq 0 \longrightarrow \mathbb{R} - \{2,5\}$$

$$\begin{vmatrix} -3 & -5+a & -3 \\ 1 & 0 & 1 \\ -2+a & -7+a & 0 \end{vmatrix} \neq 0 \longrightarrow \mathbb{R} - \{2,5\}$$

$$\begin{vmatrix} -3 & -8+a & -3 \\ 1 & 1 & 1 \\ -2+a & -7+a & 0 \end{vmatrix} \neq 0 \longrightarrow \mathbb{R} - \{2,5\}$$

Sea $T : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ la simetría de $\mathbb{R}_2[x]$ con respecto al subespacio $\text{gen}\{x - x^2\}$ $\mathbb{S}_1$ en la dirección del subespacio $\text{gen}\{1 - 2x, 1 + x^2\}$ $\mathbb{S}_2$. La matriz de T con respecto a la base canónica $\{1, x, x^2\}$ es ...

$S_1 = \{(0,1,-1)\}$ $\qquad$ $S_2 = \{(1,-2,0);(1,0,1)\}$

2.19 Sea $\mathbb{V}$ un $\mathbb{K}$-espacio vectorial y sean $\mathbb{S}_1, \mathbb{S}_2$ dos subespacios suplementarios de $\mathbb{V}$, esto es, todo vector $v \in \mathbb{V}$ se escribe de manera única como $v = v_1 + v_2$ con $v_1 \in \mathbb{S}_1$ y $v_2 \in \mathbb{S}_2$.

$$\Sigma_{\mathbb{S}_1\mathbb{S}_2}(v) = \begin{cases} v & \text{si } v \in \mathbb{S}_1, \\ -v & \text{si } v \in \mathbb{S}_2, \end{cases}$$

razón por la cual $\Sigma_{\mathbb{S}_1\mathbb{S}_2}$ de denomina *la simetría de* $\mathbb{V}$ *con respecto a* $\mathbb{S}_1$ *en la dirección de* $\mathbb{S}_2$.

$T(0,1,-1) = (0,1,-1)$ ✓

$T = \left(T(1,0,0)^T \; T(0,1,0)^T \; T(0,0,1)^T \right)$

$T(1,-2,0) = (-1,2,0)$ ✓

$T(\bar{x}) = M_T \cdot \bar{x}$

$T(1,0,1) = (-1,0,-1)$ ✓

$$\frac{1}{5}\begin{bmatrix} -3 & 0 & 0 \\ 4 & -1 & -4 \\ -4 & -2 & 1 \end{bmatrix}$$

M_T

Sea $T \in \mathcal{L}(\mathbb{R}_2[x], \mathbb{R}^3)$ y sea $[T]_B^C = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix}$ la matriz de T con respecto a las bases

$B = \left\{ \frac{1}{2}(x-1)(x-2), -x(x-2), \frac{1}{2}x(x-1) \right\}$ de $\mathbb{R}_2[x]$ y $C = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ de $\mathbb{R}^3$.

Si $S = \text{gen}\{1-x, 1+x\}$, entonces ...

$S = \{(1,-1,0), (1,1,0)\}$ (s_1, s_2)

$B = \{1 - \frac{3}{2}x + \frac{1}{2}x^2, 2x - x^2$

$B = \{(1, -\frac{3}{2}, \frac{1}{2}), (0, 2, -1), (0, -\frac{1}{2}, \frac{1}{2})\}$

$[s_1]_B = (1, 0, -1)$

$[s_2]_B = (1, 2, 3)$

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$$T([v_1]_B) = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}_C \qquad T([v_2]_B) = \begin{pmatrix} -8 \\ 7 \\ 12 \end{pmatrix}_C$$

$$C = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

$c_1 \quad c_2 \quad c_3$

$$T([v_1]_B) = 2c_1 - c_2 - 2c_3 = (3,-1,-2)$$

$$T([v_2]_B) = -8c_1 + 7c_2 + 12c_3 = (-9,21,18)$$

$$T(S) = \{(3,-1,-2),(-9,21,18)\}$$

Maximal linearly independent subset

$\{(3,-1,-2),(-9,21,18)\}$

linear independence (3,-1,-2), (-9,21,18), (-3,10,8), (-6,11,10)