En $\mathbb{R}_2[x]$ con el producto interno definido por

$\langle p, q\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$

se considera $\phi : \mathbb{R}_2[x] \to \mathbb{R}$ la funcional lineal definida por

$\phi(p) = \frac{1}{4}\int_0^4 p(x)dx.$

El único polinomio $q \in \mathbb{R}_2[x]$ tal que $\phi(p) = \langle p, q\rangle$ para todo $p \in \mathbb{R}_2[x]$ es

$$\langle p, q\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$$

$$\boxed{\begin{aligned}\langle p, q\rangle = a_0 \cdot b_0 &+ [a_0 + a_1 + a_2][b_0 + b_1 + b_2] \\ &+ [a_0 + 2a_1 + 4a_2][b_0 + 2b_1 + 4b_2]\end{aligned}}$$

(1)

<u>Veaorid</u>

$E_{\mathbb{R}_2[x]} = \{1, x, x^2\}$

$P(x) = a_0 + a_1 x + a_2 x^2$

$q(x) = b_0 + b_1 x + b_2 x^2$

$P(0) = a_0 \quad P(1) = a_0 + a_1 + a_2$

$P(2) = a_0 + 2a_1 + 4a_2$

$q(0) = b_0 \quad , \quad q(1) = b_0 + b_1 + b_2$

$q(2) = b_0 + 2b_1 + 4b_2$

$$\phi(p) = \frac{1}{4}\int_0^4 p(x)dx.$$

$$\phi(p) = \frac{1}{4}\int_0^4 (a_0 + a_1 x + a_2 x^2)dx$$

(2) $$\int_0^4 \frac{1}{4}(a_0 + a_1 x + a_2 x^2)\,dx = a_0 + 2a_1 + \frac{16a_2}{3}$$

Expanded form

$$3a_0 b_0 + 3a_1 b_0 + 5a_2 b_0 + 3a_0 b_1 + 5a_1 b_1 + 9a_2 b_1 + 5a_0 b_2 + 9a_1 b_2 + 17a_2 b_2$$

$$a_0[3b_0 + 3b_1 + 5b_2] + a_1[3b_0 + 5b_1 + 9b_2]$$

$$+ a_2[5b_0 + 9b_1 + 17b_2]$$

$$\phi(p) = \langle p, q \rangle$$

$$a_0 \mid \; 3b_0 + 3b_1 + 5b_2 = 1$$

$$a_1 \mid \; 3b_0 + 5b_1 + 9b_2 = 2$$

$$a_2 \mid \; 5b_0 + 9b_1 + 17b_2 = 16/3$$

3x + 3y + 5z =1, 3x + 5y + 9z=2, 5x + 9y + 17z = 16/3

$$\underset{b_0}{x = \frac{2}{3}}, \quad \underset{b_1}{y = -\frac{9}{2}}, \quad \underset{b_2}{z = \frac{5}{2}}$$

$$\boxed{q(x) = \frac{2}{3} - \frac{9}{2}x + \frac{5}{2}x^2}$$

Sea $Y(t)$ la solución del sistema de ecuaciones diferenciales

$$\begin{cases} y_1' = -17y_1 + 8y_2 + 22y_3 \\ y_2' = -3y_1 + 3y_2 + 4y_3 \\ y_3' = -14y_1 + 6y_2 + 18y_3 \end{cases}$$

tal que $Y(0) = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}^T$. Vale que

$y(1)$

① $\boxed{X' = AX}$

$$X' = \begin{pmatrix} -17 & 8 & 22 \\ -3 & 3 & 4 \\ -14 & 6 & 18 \end{pmatrix} X'$$

diagonalize {{-17,8,22},{-3,3,4},{-14,6,18}}

diagonalize	$\begin{pmatrix} -17 & 8 & 22 \\ -3 & 3 & 4 \\ -14 & 6 & 18 \end{pmatrix}$

$M = S.J.S^{-1}$

$$M = \begin{pmatrix} -17 & 8 & 22 \\ -3 & 3 & 4 \\ -14 & 6 & 18 \end{pmatrix} \qquad J = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

D

$r_1 = 1$

$r_2 = 1$

$r_3 = 2$

$$S = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -2 & 2 \\ 2 & 0 & 1 \end{pmatrix} \qquad S^{-1} = \begin{pmatrix} 2 & -1 & -2 \\ -5 & 2 & 6 \\ -4 & 2 & 5 \end{pmatrix}$$

P $\quad$ P^{-1}

$$Y = \begin{pmatrix} c_1 \cdot e^{r_1 t} \\ c_2 t e^{r_2 t} \\ c_3 e^{r_3 t} \end{pmatrix} = \begin{pmatrix} c_1 \cdot e^{t} \\ c_2 t e^{t} \\ c_3 e^{2t} \end{pmatrix}$$

$X(t) = PY(t)$

$$X(t) = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -2 & 2 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \cdot e^{t} \\ c_2 t e^{t} \\ c_3 e^{2t} \end{pmatrix}$$

$$\begin{cases} y_1' = -17y_1 + 8y_2 + 22y_3 \\ y_2' = -3y_1 + 3y_2 + 4y_3 \\ y_3' = -14y_1 + 6y_2 + 18y_3 \end{cases}$$

tal que $Y(0) = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}^T$. Vale que

$$Y' = \begin{pmatrix} -17 & 8 & 22 \\ -3 & 3 & 4 \\ -14 & 6 & 18 \end{pmatrix} y'$$

Y'={{-17,8,22},{-3,3,4},{-14,6,18}}*Y

$$\vec{Y}(x) = \begin{pmatrix} c_1(-e^x)(10x + 8e^x - 9) + 4c_2 e^x (x + e^x - 1) + 2c_3 e^x (6x + 5e^x - 5) \\ c_1(-e^x)(-5x + 8e^x - 8) + c_2 e^x (-2x + 4e^x - 3) + 2c_3 e^x (-3x + 5e^x - 5) \\ -2c_1 e^x (5x + 2e^x - 2) + 2c_2 e^x (2x + e^x - 1) + c_3 e^x (12x + 5e^x - 4) \end{pmatrix}$$

$x = 0$ $\qquad$ $e^0 = 1$

c_1 $\qquad$ c_2 $\qquad$ c_3

$$\vec{Y}(x) = \begin{pmatrix} c_1(-e^x)(\cancel{10x} + 8e^x - 9) + 4c_2 e^x (\cancel{x} + e^x - 1) + 2c_3 e^x (\cancel{6x} + 5e^x - 5) \\ c_1(-e^x)(\cancel{-5x} + 8e^x - 8) + c_2 e^x (\cancel{-2x} + 4e^x - 3) + 2c_3 e^x (\cancel{-3x} + 5e^x - 5) \\ -2c_1 e^x (\cancel{5x} + 2e^x - 2) + 2c_2 e^x (\cancel{2x} + e^x - 1) + c_3 e^x (\cancel{12x} + 5e^x - 4) \end{pmatrix}$$

$$Y(0) = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{Y}(x) = \begin{pmatrix} c_1(-e^x)(10x + 8e^x - 9) + 4c_2 e^x (x + e^x - 1) + 2c_3 e^x (6x + 5e^x - 5) \\ c_1(-e^x)(-5x + 8e^x - 8) + c_2 e^x (-2x + 4e^x - 3) + 2c_3 e^x (-3x + 5e^x - 5) \\ -2c_1 e^x (5x + 2e^x - 2) + 2c_2 e^x (2x + e^x - 1) + c_3 e^x (12x + 5e^x - 4) \end{pmatrix}$$

$+e \quad 1+8e \quad 4e \quad e \quad 2e \quad 1+5e$

$e \quad 8e-13 \quad e \quad 4e-5 \quad 2e \quad 5e-8$

$2e \quad 3+2e \quad 2e \quad 1+e \quad e \quad 8+5e$

$$\vec{Y}(x) = \begin{pmatrix} c_1(-e^x)(10x + 8e^x - 9) + 4c_2 e^x (x + e^x - 1) + 2c_3 e^x (6x + 5e^x - 5) \\ c_1(-e^x)(-5x + 8e^x - 8) + c_2 e^x (-2x + 4e^x - 3) + 2c_3 e^x (-3x + 5e^x - 5) \\ -2c_1 e^x (5x + 2e^x - 2) + 2c_2 e^x (2x + e^x - 1) + c_3 e^x (12x + 5e^x - 4) \end{pmatrix}$$

$$Y(1) = \begin{pmatrix} e + 8e^2 + 4e^2 + 2e + 10e^2 \\ 8e^2 - 13e + 4e^2 - 5e + 10e^2 - 16e \\ 6e + 4e^2 + 2e + 2e^2 + 5e^2 + 8e \end{pmatrix} = \begin{pmatrix} 22e^2 + 3e \\ 22e^2 - 34e \\ 11e^2 + 16e \end{pmatrix}$$

Sea $Q:\mathbb{R}^3 \to \mathbb{R}$ la forma cuadrática definida por $Q(x) = x^T \begin{bmatrix} 14 & 4 & 0 \\ 4 & 12 & -4 \\ 0 & -4 & 10 \end{bmatrix} x$

$Q(x) = 12\|x\|^2$ si, y solo si,

$$A = \begin{bmatrix} 14 & 4 & 0 \\ 4 & 12 & -4 \\ 0 & -4 & 10 \end{bmatrix}$$

diagonalize {{14,4,0},{4,12,-4},{0,-4,10}}

$$M = S.J.S^{-1}$$

$$M = \begin{pmatrix} 14 & 4 & 0 \\ 4 & 12 & -4 \\ 0 & -4 & 10 \end{pmatrix}$$

$$J = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix}$$

D

$$S = \begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} -\frac{1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \\ -\frac{2}{9} & -\frac{2}{9} & \frac{1}{9} \end{pmatrix}$$

$$Q(y) = y^T \cdot D \cdot y$$

$$y^T \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix} y$$

$$Q(x) = 12\|x\|^2$$

$$\|x\|^2 = x^T \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot x$$

$$\|y\|^2 = \|x\|^2 = y^T \cdot I \cdot y$$

$$Q(x) = Q(x)$$

$$12\|x\|^2 = y^T \cdot D \cdot y$$

Teorema de Rayleigh: sea la forma cuadrática $Q(x) = x^T A x$, con A simétrica. Se verifica:

$$\lambda_{min}(A) \leq \frac{Q(x)}{\|x\|^2} \leq \lambda_{max}(A)$$

Sea extremar una forma cuadrática $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) = x^T A x$ (A simétrica), sujeto a la restricción $|x| = \alpha$. El máximo de f es $\lambda_{max}(A).\alpha^2$ y se alcanza en $M = \{x \in S_{\lambda_{max}}(A) : |x| = \alpha\}$. El mínimo de f es $\lambda_{min}(A).\alpha^2$ y se alcanza en $m = \{x \in S_{\lambda_{min}}(A) : |x| = \alpha\}$.

Sea $Q : \mathbb{R}^3 \to \mathbb{R}$ la forma cuadrática definida por $Q(x) = x^T \begin{bmatrix} 14 & 4 & 0 \\ 4 & 12 & -4 \\ 0 & -4 & 10 \end{bmatrix} x$

$Q(x) = 12\|x\|^2$ si, y solo si,

$$Q(x) = 12\|x\|^2$$

$$\frac{Q(x)}{\|x\|^2} = 12$$

$$\lambda_{min}(A) \leq \frac{Q(x)}{\|x\|^2} \leq \lambda_{max}(A)$$

$$\lambda_{min} \leq 12 \leq \lambda_{max}$$

$$J = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix}$$

D

$$M = \begin{pmatrix} 14 & 4 & 0 \\ 4 & 12 & -4 \\ 0 & -4 & 10 \end{pmatrix}$$

$$S = \begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 18 \end{pmatrix}$$

D

$$S^{-1} = \begin{pmatrix} -\frac{1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} \\ -\frac{2}{9} & -\frac{2}{9} & \frac{1}{9} \end{pmatrix}$$

$Sol = \{(2, -1, 2)$

c. $x \in \text{gen}\left\{\begin{bmatrix}2\\-1\\2\end{bmatrix}\right\}.$

El mínimo de $\frac{1}{5}\left(29x_1^2+24x_1x_2+36x_2^2\right)$ sujeto a la restricción $-\frac{1}{25}\left(23x_1^2-72x_1x_2+2x_2^2\right)=1$ es

$f(x)$

$R(x)=c$

Sea extremar una forma cuadrática $f:\mathbb{R}^n\to\mathbb{R}$, $f(x)=x^TAx$ (A simétrica), sujeto a la restricción $x^TBx=\alpha^2$, y sea B definida positiva tal que $B=P_BD_BP_B^T$. Mediante un cambio de variable $y=\sqrt{D_B}P_B^Tx$ ($x=\sqrt{D_B^{-1}}P_By$), esto es equivalente a extremar $g(y)=y^T\left(\sqrt{D_B^{-1}}P_B^TAP_B\sqrt{D_B^{-1}}\right)y$ sujeto a la restricción $|y|=\alpha$. Entonces: $\max g(y)=\max f(x)$, y $\min g(y)=\min f(x)$. Los x en donde se alcanza ese extremo se hallan realizando la cuenta $x=P_By$.

$$B=P_BD_BP_B^T$$

diagonalize $\begin{pmatrix}-\frac{23}{25} & \frac{36}{25}\\ \frac{36}{25} & -\frac{2}{25}\end{pmatrix}$

$\lambda_{min}=1$, $V_{min}=\left(\frac{3}{4},1\right)$

$V_{min}=(3,4)$

$V_{min}=\left(\frac{3}{5},\frac{4}{5}\right)$

$$B=\begin{pmatrix}-23/25 & 36/25\\ 36/25 & -2/25\end{pmatrix}$$

$$M=S.J.S^{-1}$$

$$S=\begin{pmatrix}-\frac{4}{3} & \frac{3}{4}\\ 1 & 1\end{pmatrix}\quad J=\begin{pmatrix}-2 & 0\\ 0 & 1\end{pmatrix}\quad S^{-1}=\begin{pmatrix}-\frac{12}{25} & \frac{9}{25}\\ \frac{12}{25} & \frac{16}{25}\end{pmatrix}$$

$V_{max} = \left(\frac{3}{5}, \frac{4}{5}\right)$

$V_{min} = \left\{\left(\frac{3}{5}, \frac{4}{5}\right)\right\} = k\left(\frac{3}{5}, \frac{4}{5}\right)$

y sujeto a la restricción $|y| = \alpha$.

$-\frac{1}{25}\left(23x_1^2 - 72x_1x_2 + 2x_2^2\right) = 1$ α^2

$X = P \cdot y$

$P^T \cdot X = P^T P y \qquad y = P^T X$

$-\left(23x_1^2 - 72x_1x_2 + 2x_2^2\right) = 25\alpha^2$

$\begin{pmatrix} -\frac{12}{25} & \frac{9}{25} \\ \frac{12}{25} & \frac{16}{25} \end{pmatrix} \begin{pmatrix} k\frac{3}{5} \\ k\frac{4}{5} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{4k}{5} \end{pmatrix} = y$

$|y| = \alpha \qquad \alpha^2 = 1 \;\; \alpha = 1$

$|y| = 0^2 + \left(\frac{4}{5}k\right)^2 = 1^2$

$\frac{16}{25}k^2 = 1^2, \quad k^2 = \frac{25}{16}$

$k = \pm\frac{5}{4} \qquad y_1 = (0, 1)$

$y_2 = (0, -1)$

$$X = P \cdot y$$

$$S = \begin{pmatrix} -\frac{4}{3} & \frac{3}{4} \\ 1 & 1 \end{pmatrix} \rightsquigarrow P = \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix}$$

$$x_1 = P \cdot y_1 = \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$

$$x_2 = P \cdot y_2 = \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$

$$\frac{1}{5}\left(29x_1^2 + 24x_1x_2 + 36x_2^2\right)$$

$$f(x_1) = \frac{1}{5}\left[29\,\frac{9}{25} + 24\,\frac{3 \cdot 4}{25} + 36 \cdot \frac{16}{25}\right]$$

$$\frac{1}{125}\left[29 \cdot 9 + 24 \cdot 12 + 36 \cdot 16\right]$$

$$= \frac{1125}{125} = 9$$

$$f(x_2) = 9$$

$$\min\left\{\frac{1}{5}\left(29x^2 + 24xy + 36y^2\right) \,\middle|\, \frac{1}{25}\left(-23x^2 + 72xy - 2y^2\right) = 1\right\} = 9 \text{ at } (x, y) = \left(-\frac{3}{5}, -\frac{4}{5}\right)$$

$$\min\left\{\frac{1}{5}\left(29x^2 + 24xy + 36y^2\right) \,\middle|\, \frac{1}{25}\left(-23x^2 + 72xy - 2y^2\right) = 1\right\} = 9 \text{ at } (x, y) = \left(\frac{3}{5}, \frac{4}{5}\right)$$

Se consideran los siguientes subespacios de $\mathbb{R}_2[x]$:

$\mathbb{S}_1 = \text{gen}\,\{4 - x + x^2,\ 2 - 2x + x^2\}$ y $\mathbb{S}_2 = \text{gen}\,\{1 + 3x - 2x^2,\ 2 + 2x - 2x^2\}$.

Vale que $S_1 \cap S_2$

$$\vec{E}\,\mathbb{R}_2[x] = \{1, x, x^2\}$$

$$S_1 = \{(4,-1,1);\ (2,-2,1)\}$$

$$S_2 = \{(1,3,-2);\ (2,2,-2)\}$$

$\underline{S_1 \cap S_2}$ $\quad v \in S_1 \cap S_2$

$$v \in S_1 \ \wedge\ v \in S_2$$

$$\boxed{v = a(4,-1,1) + b(2,-2,1)}$$

$$\boxed{v = c(1,3,-2) + d(2,2,-2)}$$

a*(4,-1,1) + b*(2,-2,1) = c*(1,3,-2) + d*(2,2,-2)

$$b = -\frac{5a}{2}, \quad c = \frac{5a}{2}, \quad d = -\frac{7a}{4}$$

$$v = a(4,-1,1) - \frac{5}{2}a(2,-2,1)$$

$$= (4a - 5a,\ -a + 5a,\ a - \frac{5}{2}a)$$

$$S_1 \cap S_2 = \{(-a, 4a, -\frac{3}{2}a)\}$$

$$S_1 \cap S_2 = \{(-2, 8, -3)\}$$

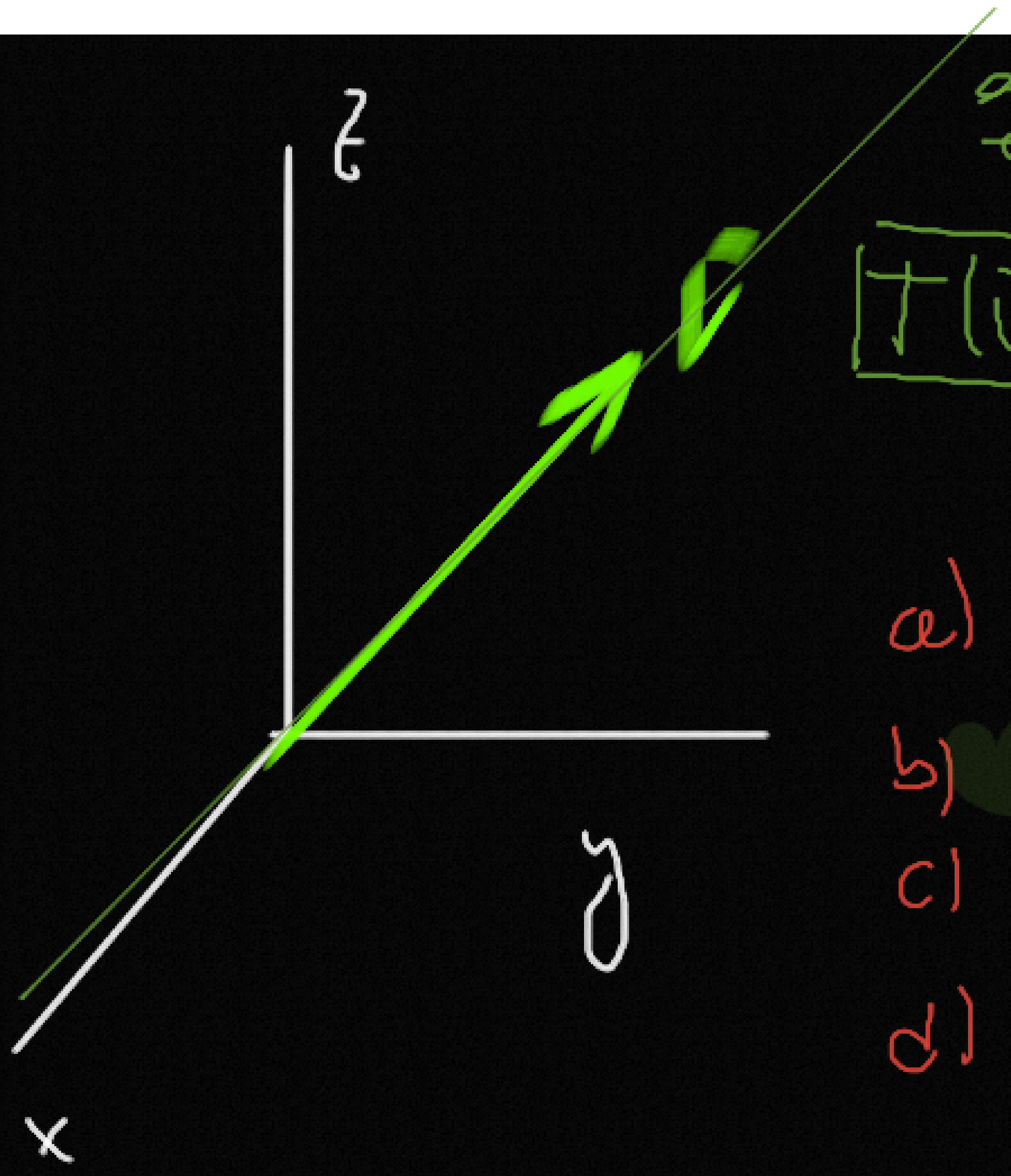
$S_1 \cap S_2 = \{(-2, 8, -3)\}$

c. $\mathbb{S}_1 \cap \mathbb{S}_2 = \text{gen}\{2 - 8x + 3x^2\}$.

$S_1 \cap S_2 = \text{gen}\{-2 + 8x - 3x^2\}$

LD de sol prop

Sea $T \in \mathcal{L}(\mathbb{R}^3)$ la transformación lineal definida por $T(x) = Ux$, donde $U = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ -2 & -1 & 2 \end{bmatrix}$.



$\boxed{T(\bar{v}) = \bar{v}}$

a) r $[1 \quad 1 \quad 1]^T$.

b) r $[1 \quad 1 \quad -1]^T$.

c) r $[-1 \quad 1 \quad 1]^T$.

d) r $[1 \quad -1 \quad 1]^T$.

Input

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Result

$$\begin{pmatrix} \frac{1}{3} \\ \frac{5}{3} \\ -\frac{1}{3} \end{pmatrix}$$

a)

Input

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Result

$$\begin{pmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ -\frac{5}{3} \end{pmatrix}$$

b)

Input

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Result

$$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

c) ✓

Input

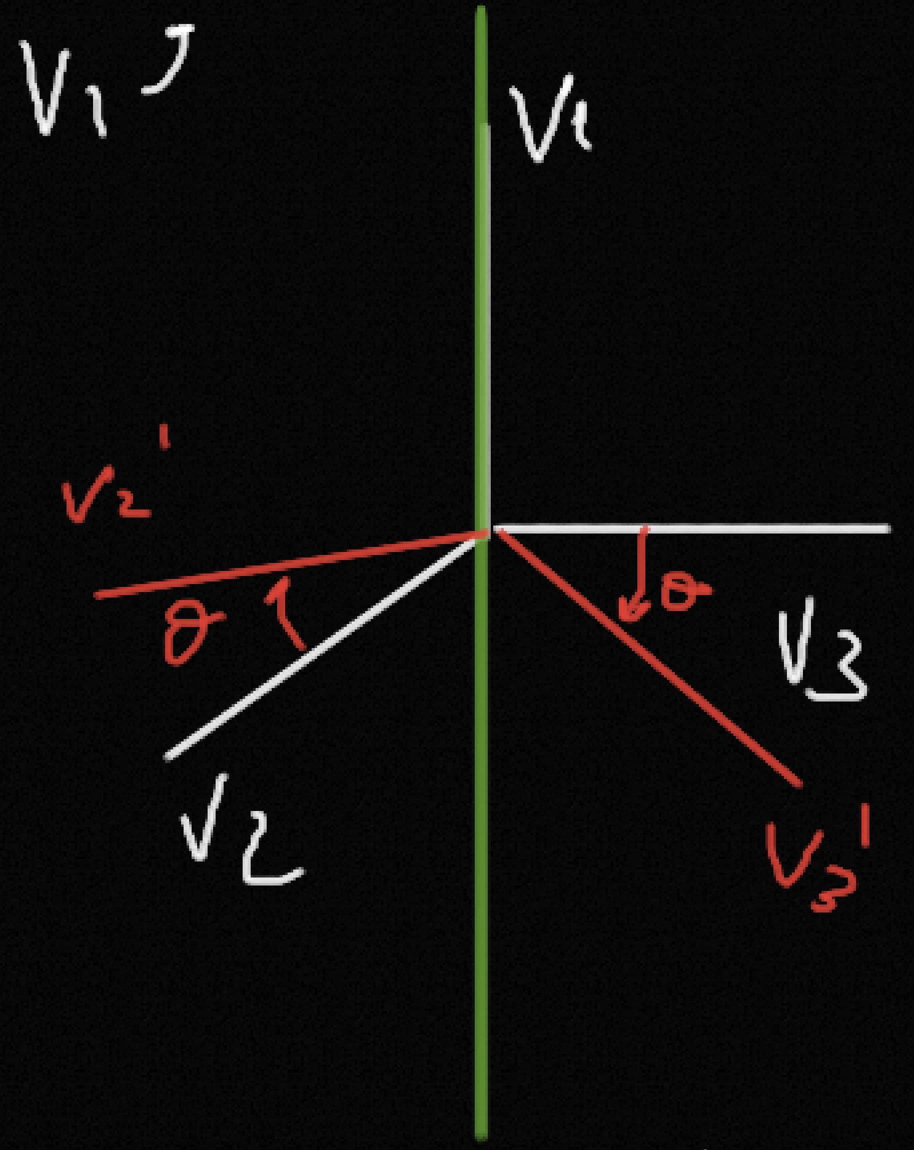
$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Result

$$\begin{pmatrix} \frac{5}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

d)

$[1 \quad 1 \quad -1]^T$. BON $\quad \bar{V}\cdot\bar{V}_1 = 0$



(1,1,-1).(a,b,c)=0

NATURAL LANGUAGE

Input

$\{1, 1, -1\}.\{a, b, c\} = 0$

Result

$a + b - c = 0$

$C = (a+b) \qquad S_\perp = \{(a, b, a+b)\}$

$S\perp = \{(1,0,1); (0,1,1)\}$

$B = \{(1,1,-1)\ (1,0,1); (0,1,1)\}$

gram schmidt {{1,1,-1},{1,0,1},{0,1,1}}

NATURAL LANGUAGE $\quad$ MATH INPUT

Input

$$\text{Orthogonalize}\left[\begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}\right]$$

Result

$$\begin{array}{cl} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} \end{pmatrix} & \begin{matrix} V_1 \\ V_2 \\ V_3 \end{matrix} \end{array}$$

BON

$$\begin{array}{c} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} \end{pmatrix} \begin{array}{l} V_1 \\ V_2 \\ V_3 \end{array} \end{array} \qquad U = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ -2 & -1 & 2 \end{bmatrix}$$

$$T(v_1) = V_1 = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$$T(v_2) = U \cdot V_2 = V_2' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$T(V_3) = U \cdot V_3 = V_3' = \begin{pmatrix} -\frac{5}{3\sqrt{6}} \\ \frac{5}{3\sqrt{6}} \\ \frac{\sqrt{\frac{2}{3}}}{3} \end{pmatrix}$$

Input

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Result

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

Expanded form

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$$

Result

$$\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{6}\sqrt{6}\,(-1) \\ \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \end{pmatrix}$$

Expanded form

$$\begin{pmatrix} -\frac{5}{3\sqrt{6}} \\ \frac{5}{3\sqrt{6}} \\ \frac{\sqrt{\frac{2}{3}}}{3} \end{pmatrix}$$

Dimensions

Sea $\Sigma : \mathbb{R}^3 \to \mathbb{R}^3$ la transformación lineal que transforma el triángulo de vértices $\begin{bmatrix}2 & -6 & 3\end{bmatrix}^T, \begin{bmatrix}6 & 3 & 2\end{bmatrix}^T, \begin{bmatrix}3 & -2 & -6\end{bmatrix}^T$ en el triángulo de vértices $\begin{bmatrix}-5 & 8 & -4\end{bmatrix}^T, \begin{bmatrix}13 & -11 & 9\end{bmatrix}^T, \begin{bmatrix}10 & -16 & 1\end{bmatrix}^T$, respectivamente. Vale que

$$T(2,-6,3) = (-5,8,-4)$$

$$T(6,3,2) = (13,-11,9)$$

$$T(3,-2,-6) = (10,-16,1)$$

$$B = \{\underbrace{\begin{bmatrix}2 & -6 & 3\end{bmatrix}^T}_{b_1}, \underbrace{\begin{bmatrix}6 & 3 & 2\end{bmatrix}^T}_{b_2}, \underbrace{\begin{bmatrix}3 & -2 & -6\end{bmatrix}^T}_{b_3}\}$$

$$C = \{\underbrace{\begin{bmatrix}-5 & 8 & -4\end{bmatrix}^T}_{c_1}, \underbrace{\begin{bmatrix}13 & -11 & 9\end{bmatrix}^T}_{c_2}, \underbrace{\begin{bmatrix}10 & -16 & 1\end{bmatrix}^T}_{c_3}\}$$

$$[T]_B^C = \left[T(b_1)_C \quad T(b_2)_C \quad T(b_3)_C \right]$$

$$[b_1]_B = (1,0,0) \qquad [T(b_1)]_C = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

La respuesta correcta es: Σ es la simetría de $\mathbb{R}^3$ con respecto al subespacio $\underbrace{\{x \in \mathbb{R}^3 : x_1 + x_2 - x_3 = 0\}}_{S_1}$ en la dirección del subespacio $\underbrace{\operatorname{gen}\left\{\begin{bmatrix}1 & -2 & 1\end{bmatrix}^T\right\}}_{S_2}$.

$$S_1 = \{\underbrace{(1,0,1)}_{v_1}, \underbrace{(0,1,1)}_{v_2}\} \qquad S_2 = \{\underbrace{(1,-2,1)}_{v_3}\}$$

$$\Sigma_{S_1 S_2} = \begin{cases} v & \text{si } v \in S_1 \\ -v & \text{si } v \in S_2 \end{cases}$$

$$a*(2,-6,3) + b*(6,3,2) + c*(3,-2,-6) = (1,0,1)$$

$$a = \frac{5}{49}, \quad b = \frac{8}{49}, \quad c = -\frac{3}{49}$$

$$T[a*(2,-6,3) + b*(6,3,2) + c*(3,-2,-6)] = T(1,0,1)$$

$$T(1,0,1) = (1,0,1)$$

$$T[a*(2,-6,3) + b*(6,3,2) + c*(3,-2,-6)] = T(0,1,1)$$

$$a = -\frac{3}{49}, \quad b = \frac{5}{49}, \quad c = -\frac{8}{49}$$

$$T(0,1,1) = (0,1,1)$$

$$T[a*(2,-6,3) + b*(6,3,2) + c*(3,-2,-6)] = T(1,-2,1)$$

$$a = \frac{17}{49}, \quad b = \frac{2}{49}, \quad c = \frac{1}{49}$$

$$T(1,-2,1) = (-1,2,-1)$$

$$T(2,-6,3) = (-5,8,-4)$$

$$T(6,3,2) = (13,-11,9)$$

$$T(3,-2,-6) = (10,-16,1)$$

$T(1,0,1) = (1,0,1)$
$T(0,1,1) = (0,1,1)$ $\Big\}$ S_1 ✓

$T(1,-2,1) = (-1,2,-1)$ $\}$ S_2 ✓

$$\overset{S_1}{S_1 = \{\underset{v_1}{(1,0,1)}, \underset{v_2}{(0,1,1)}\}} \quad \overset{S_2}{S_2 = \{\underset{v_3}{(1,-2,1)}\}}$$

$$\Sigma_{S_1 S_2} = \begin{cases} v & \text{si } v \in S_1 \checkmark \\ -v & \text{si } v \in S_2 \checkmark \end{cases}$$