Ejercicio 1

En $\mathbb{R}^4$

$$S_1 = gen\left\{\begin{pmatrix}1\\2\\2\\1\end{pmatrix}\begin{pmatrix}0\\3\\0\\-1\end{pmatrix}\right\} \quad S_2 = gen\left\{\begin{pmatrix}1\\0\\1\\2\end{pmatrix}\begin{pmatrix}1\\2\\1\\0\end{pmatrix}\right\}$$

$$T \,/\, S_1 \oplus T = S_2 \oplus T = \{x \in \mathbb{R}^4 : x_1 - x_2 + x_3 - x_4 = 0\} = S$$

Pasos a seguir

1º) $S_1 \subset S \wedge S_2 \subset S$, de lo contrario no existe T.

2º) Verificar que $\dim(S) = 3$

3º) Como $S_1 \oplus T = S \wedge \dim(S_1) = 2$; $\dim(S) = 3$

$\Rightarrow \dim(T) = 1$ (también se cumple para S_2)

4º) Alcanza con tomar $T = gen\{v_i\}$ $(v_i \neq 0)$ /

$v_i \in S \wedge v_i$ li con los de $S_1 \wedge$ de S_2

así $S_1 + T \subset S \wedge S_1 \cap T = \{0_{\mathbb{R}^4}\}$

$S_2 + T \subset S \wedge S_2 \cap T = \{0_{\mathbb{R}^4}\}$

$\wedge \dim(S_1 + T) = 3 = \dim(S)$

Ejercicio 2

$$A = \begin{pmatrix}3 & 2\\-5 & 1\end{pmatrix} \quad Y : \mathbb{R} \to \mathbb{R}^2 \quad Y' = AY \quad Y(0) = \begin{pmatrix}2\\3\end{pmatrix} \text{. Hallar } Y\left(\tfrac{\pi}{6}\right)$$

AVAS de A : $\lambda_1 = 2+3i$ $\lambda_2 = 2-3i$

$$S_{\lambda_1} = gen\left\{\begin{pmatrix}2\\-1+3i\end{pmatrix}\right\} \quad S_{\lambda_2} = gen\left\{\begin{pmatrix}2\\-1-3i\end{pmatrix}\right\}$$

$$\phi(t) = e^{(2+3i)t}\begin{pmatrix}2\\-1+3i\end{pmatrix} = \left(e^{2t}\cos 3t + i e^{2t}\operatorname{sen} 3t\right)\begin{pmatrix}2\\-1+3i\end{pmatrix}$$

$$\phi(t) = \underbrace{\begin{pmatrix}2e^{2t}\cos 3t\\-e^{2t}\cos 3t - 3e^{2t}\operatorname{sen} 3t\end{pmatrix}}_{Y_1(t)} + i\underbrace{\begin{pmatrix}2e^{2t}\operatorname{sen} 3t\\-e^{2t}\operatorname{sen} 3t + 3e^{2t}\cos 3t\end{pmatrix}}_{Y_2(t)}$$

$$\Rightarrow Y(t) = A\begin{pmatrix} 2e^{2t}\cos 3t \\ -e^{2t}\cos 3t - 3e^{2t}\operatorname{sen} 3t \end{pmatrix} + B\begin{pmatrix} 2e^{2t}\operatorname{sen} 3t \\ 3e^{2t}\cos 3t - e^{2t}\operatorname{sen} 3t \end{pmatrix} \quad A, B \in \mathbb{R}$$

$Y(t): \mathbb{R} \to \mathbb{R}^2$

$$Y(0) = A\begin{pmatrix} 2 \\ -1 \end{pmatrix} + B\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \therefore \quad \begin{matrix} A = 1 \\ B = \frac{4}{3} \end{matrix}$$

$$Y(t) = \begin{pmatrix} 2e^{2t}\cos 3t \\ -e^{2t}\cos 3t - 3e^{2t}\operatorname{sen} 3t \end{pmatrix} + \frac{4}{3}\begin{pmatrix} 2e^{2t}\operatorname{sen} 3t \\ 3e^{2t}\cos 3t - e^{2t}\operatorname{sen} 3t \end{pmatrix}$$

$$Y\left(\frac{\pi}{6}\right) = e^{\frac{\pi}{3}}\begin{pmatrix} \frac{8}{3} \\ -\frac{10}{3} \end{pmatrix}$$

Ejercicio 3

$A \in \mathbb{R}^{2\times 3}$

$(1\ 1\ 1)^T \in Nul(A)$ $\quad v = (-4\ 0\ 4)^T$ ave de A^TA $/ Av = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$\max_{\|x\|=1} \|Ax\| = \sqrt{2}$$

$$\sigma_{Máx} = \sqrt{2}$$

En $\quad A\begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ con $\|v\| = 4\sqrt{2}$

$$A\frac{\|v\|}{\|v\|}\begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow A\begin{pmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{4\sqrt{2}}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{3}{4\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}}_{u}$$

$$\|u\| = \frac{5}{4\sqrt{2}} \Rightarrow \frac{u}{\|u\|} = u_2 = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \text{ con } \sigma_2 = \frac{5}{8}\sqrt{2} < \sigma_{Máx} = \sqrt{2}$$

$$\therefore \sigma_{Máx} = \sqrt{2} \text{ con } u_1 = \begin{pmatrix} -\frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$$

DVS reducida de A :

$$A = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \frac{5}{8}\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$\perp$ a $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ y de norma 1

Ejercicio 4

$\max Q_1(x)$ sujeto a $Q_2(x)=1$

$Q_1(x) = x^T\begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}x \qquad Q_2(x) = x^T\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}x$

En $Q_2(x)$ con $A = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ con avas $\begin{cases} \lambda_1 = \frac{3}{2} \rightarrow v_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \\ \lambda_2 = \frac{1}{2} \rightarrow v_2 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \end{cases}$

cambio de variable $X = PY$ con $P = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$

$$\tilde{Q}_2(Y) = \frac{3}{2}y_1^2 + \frac{1}{2}y_2^2 = \underbrace{\left(\frac{y_1}{\sqrt{\frac{2}{3}}}\right)}_{z_1}^2 + \underbrace{\left(\frac{y_2}{\sqrt{2}}\right)}_{z_2}^2 = 1$$

$$\Rightarrow Y = \underbrace{\begin{pmatrix} \sqrt{\frac{2}{3}} & 0 \\ 0 & \sqrt{2} \end{pmatrix}}_{M}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \qquad X = PMZ$$

En $Q_1(x)$ es $\tilde{Q}_2(Z) = Z^T M^T P^T \begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix} PMZ = Z^T \begin{pmatrix} 5/3 & 0 \\ 0 & 3 \end{pmatrix} Z$

$\therefore \max\limits_{\|Z\|=1} \tilde{Q}(Z) = 3$ y se realiza en $Z = \pm\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

y $X_{Máx} = \pm PM\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \pm\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ 0 & \sqrt{2} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \pm\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}\begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix} = \pm\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

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